MATH 341 — FALL 2009
ASSIGNMENT 10

Due December 11, 2009

Do Problem 9.1 and any four of the remaining 7 problems: 9.2 – 9.8. You must submit 5 problems in all.

In the following Γ is the circle in the Euclidean plane centered at $O = (0,0)$ of radius 1: $\Gamma = \{(x,y) | x^2 + y^2 = 1\}$.

9.1 Let $A = (0,0)$ and $B = (0,1/4)$, and let ℓ be the diameter of Γ cut out by the x-axis.

(a) Find the Poincaré length $d_p(A,B)$.

(b) Find the coordinates of the point M on the segment $\overline{AB}$ that represents its midpoint in the Poincaré model.

9.2 (a) In a right triangle with standard notation and right angle at C, show that

$$\tan(A) = \frac{\tanh(a)}{\tanh(b)}.$$

(b) Deduce that in an isosceles triangle with base b and side a, summit angle at B and one base angle at A:

$$\tanh(a)\cos\left(\frac{B}{2}\right) = \tan(A)\sinh\left(\frac{b}{2}\right)$$
$$\sin(A)\cosh\left(\frac{b}{2}\right) = \cos\left(\frac{B}{2}\right)$$

(HINT: Drop the altitude to the base.)

9.3 In a right triangle $\triangle ABC$ with right angle at C (and standard notation), show that

$$\sin(K) = \frac{\sinh(a)\sinh(b)}{1 + \cosh(a)\cosh(b)}$$

where K = the area = $\triangle(ABC)$.

9.4 (a) Let h denote the length of the altitude from vertex B. Show that

$$\sinh b \sinh h = S \sinh a \sinh b \sinh c,$$

where S is the constant ratio in the Hyperbolic Law of Sines.

(b) Let $H = \frac{1}{2}\sinh b \sinh h$. Show that $2H = \sin A \sinh b \sinh c$.

9.5 An right isosceles triangle in the hyperbolic plane has leg length 1.4 units. Find the angles, the hypotenuse and the hyperbolic area.

9.6 If $\triangle ABC$ is an equilateral triangle with sides of length a and angle α, show that the sides and angles are related by the equations:

$$\cos(\alpha) = \frac{\cosh(a)}{1+\cosh(a)} \quad \text{and} \quad \cosh(a) = \frac{\cos(\alpha)}{1-\cos(\alpha)}.$$

9.7 Let $\triangle ABC$ be an equilateral triangle in the hyperbolic plane. Fill in the following table. Make your calculations correct to 3 decimal places.

Side	Angle (degrees & radians)	Area (radians)
0.250		
1.000		
2.000		
	5° or $\pi/36$ radians	
	1° or $\pi/180$ radians	
		$3\pi/4 \approx 2.356$

9.8 Let a, b, and c be the lengths of the sides, base, and summit, respectively, of a Saccheri quadrilateral.

(a) If d is the length of the segment joining the midpoints of the base and summit, use the diagonal from the midpoint of the base to the summit angle to show

$$\cosh(d) = \frac{\cosh(a)\cosh(b/2)}{\cosh(c/2)}.$$

(b) Show that $\cosh(c) = \cosh^2(a)\cosh(b) - \sinh^2(a)$

(c) Use the diagonal of the Saccheri quadrilateral to prove that each summit angle θ satisfies

$$\cos(\theta) = \frac{sinh(a)\cosh(a)(\cosh(b)-1)}{\sinh(c)}.$$

(d) Find c, d, and θ if

i. $a = 1$ and $b = 2$,

ii. $a = 2$ and $b = 4$.