MATH 6118-090
Non-Euclidean Geometry

Exercise Set #2

1. Suppose f is an isometry and suppose there exist two distinct points P and Q such that $f(P) = P$ and $f(Q) = Q$. Show that f is either the identity or a reflection.
2. Prove that if a line $\ell_1 \neq \ell$ is sent to itself under a reflection through ℓ, then ℓ_1 and ℓ intersect at right angles.
3. Suppose that f and g are two isometries such that $f(A) = g(A)$ and $f(B) = g(B)$, and $f(C) = g(C)$ for some nondegenerate triangle $\triangle ABC$. Show that $f = g$. That is, show that $f(P) = g(P)$ for any point P.
4. Prove the Star Trek lemma for an acute angle for which the center O is outside the angle.
5. (**Bow Tie Lemma**) Let A, A', B and C lie on a circle, and suppose $\angle BAC$ and $\angle BA'C$ subtend the same arc. Show that $\angle BAC \cong \angle BA'C$.

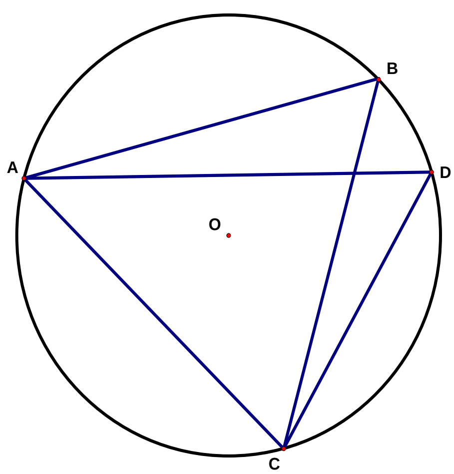

Figure 1

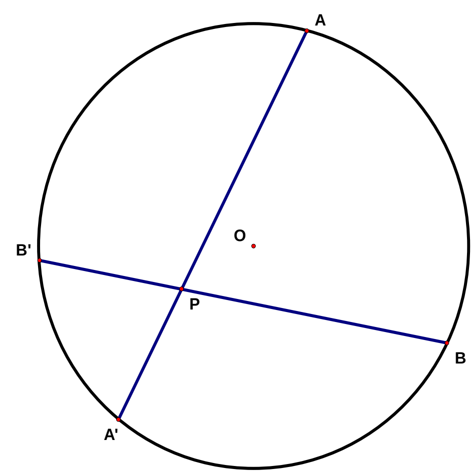

Figure 2

6. In Figure 1, if $|AB| = |AC| = |BC|$, what is the angle at D?
7. Suppose that two lines intersect at P inside a circle and meet the circle at A and A' and at B and B', as shown in Figure 2. Let α and β be the measures of the arcs $\widehat{A'B'}$ and $\widehat{AB}$ respectively. Prove that

$$\angle APB = \frac{\alpha + \beta}{2}.$$

8. Suppose an angle α is defined by two rays which intersect a circle at four points. Suppose the angular measure of the outside arc it subtends is β and the angular measure of the inside arc it subtends is γ .(So in Figure 3, $\angle AOB = \beta$ and $\angle A'OB' = \gamma$.) Show

$$\alpha = \frac{\beta - \gamma}{2}$$

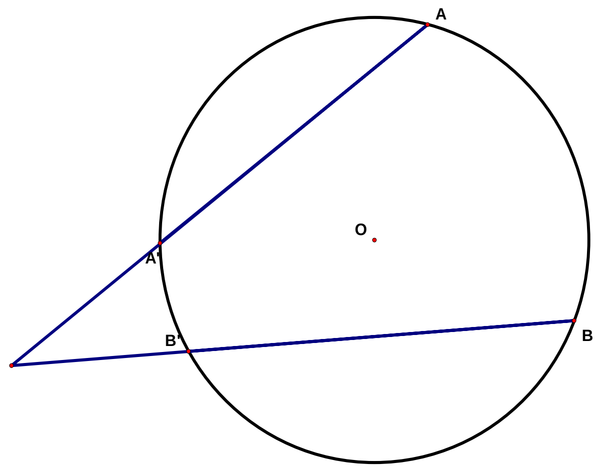

Figure 3