MATH 6118-090
Non-Euclidean Geometry

Exercise Set #7

1. In the upper half plane model, $\mathscr{H}$, carefully draw the asymptotic triangle with vertices i, $1 + i$, and 1. Is the map

$$\gamma = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

an isometry of $\mathscr{H}$? In the same diagram, carefully draw the image of the asymptotic triangle under the action of γ.

2. In the upper half plane model, $\mathscr{H}$, carefully draw the asymptotic triangle with vertices i, $-1 + i$, and $1 + i$. In the same diagram, carefully draw the image of this triangle under the isometry

$$\gamma = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

3. Let $P = \dfrac{8+i}{13}$, $Q = \dfrac{13+i}{20}$, and $\gamma = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$. What are γP and γQ? Sketch P, Q and their images. Is γ an isometry? Why? Use all of this information to find the distance between P and Q in $\mathscr{H}$.

4. Let $P = 2 + 4i$ and $Q = \dfrac{6+4i}{3}$ be two points in the upper half plane, $\mathscr{H}$. Let

$$\gamma = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}.$$

What are γP and γQ? What is the Poincaré distance from P to Q in $\mathscr{H}$.

5. Suppose that T is a fractional linear transformation such that $T(1) = 1$, $T(0) = 0$, and $T(\infty) = \infty$. Prove that T is the identity map. That is, show that $T(z) = z$ for all z.

6. Show that the dilation $\delta_\lambda(z) = \lambda z$ is an isometry of $\mathscr{H}$. Find an isometry which sends $a + bi$ to I by composing a dilation δ_λ and a horizontal translation τ_r for some λ and r. (**Hint**: You have to find the specific λ and r that will work for this example.)