

# MATH 6118-090

## Non-Euclidean Geometry

### Exercise Set #9

1. Consider the doubly asymptotic triangle  $\triangle AMN$  in  $\mathcal{H}$  where  $A = \frac{8+i}{5}$ ,  $M = \frac{5}{3}$ , and  $N = 2$ .  
What is the image of  $\triangle AMN$  under the isometry

$$\gamma = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}?$$

Use this to find the hyperbolic area of  $\triangle AMN$ .

2. Draw the asymptotic triangle  $\triangle ABM$  in  $\mathcal{H}$  with  $A = i$ ,  $B = i\sqrt{3}$ , and  $M = 1$ . What is the Poincaré length  $d(A, B)$ ? What is the area of  $\triangle ABM$ ?
3. In  $\mathcal{H}$  we have a triangle with angles  $A = 10^\circ$ ,  $B = 20^\circ$ , and  $C = 40^\circ$ . Find the sides of this triangle.
4. In  $\mathcal{H}$  we have a triangle with sides  $a = 3$ ,  $b = 4$ , and  $c = 5$ . Find the angles in this triangle. Is it a right triangle?
5. What is the area of the triangle with sides 3, 4, and 5?
6. Given a triangle in  $\mathcal{H}$  with sides 2 and 4 and included angle of  $30^\circ$ , what is the third side and what are the other two angles?
7. Given a triangle in  $\mathcal{H}$  with angles  $2^\circ$  and  $4^\circ$  and included side of 7, what is the third angle, the other two sides, and the area of this triangle?
8. Let  $\triangle ABC$  be a right triangle with the right angle at  $C$ . Prove that
- $$\cos A = \cosh a \sin B.$$
9. Let  $\triangle ABC$  be a right triangle with the right angle at  $C$ . Prove that
- $$\cot A \cot B = \cosh c.$$
10. Find all sides of the isosceles right triangle with angles  $A = B = 30^\circ$ .