MATH 6118-090
Non-Euclidean Geometry

Exercise Set #5

A *parallelogram* is defined to be a quadrilateral in which the lines containing opposite sides are non-intersecting.

1. Prove that in Euclidean geometry, a quadrilateral is a parallelogram if and only if opposite sides are congruent. Show with a generic example that in hyperbolic geometry, the opposite sides of a parallelogram need not be congruent.

NOTE: For the remainder of this problem, the geometry is *hyperbolic*.

2. Given $\square ABCD$ with opposite sides congruent, prove that opposite angles are congruent and that the lines containing opposite sides are hyperparallel. Such a quadrilateral is called a *symmetric parallelogram*.
3. For a symmetric parallelogram $\square ABCD$ prove that the diagonals have the same midpoint, M. Show that M is also the midpoint of the common perpendicular of both pairs of hyperparallel opposite sides.
4. Show that the diagonals are perpendicular if and only if all four sides are congruent, and in that case, $\square ABCD$ has an inscribed circle with center M.
5. Show that the diagonals are congruent if and only if all four angles are congruent; however in that case, show that all four sides need not be congruent.