ASSIGNMENT 2

January 28, 2008

1. Suppose that *parallelogram* has been defined as a quadrilateral with two pairs of parallel sides and the following theorem is the first property of parallelograms to be deduced. There is something wrong with the proof presented here. Find the error and correct the proof.

   Theorem: The diagonals of a parallelogram bisect each other.

   Proof: *Let $\square ABCD$ be a parallelogram with diagonals AC and BD intersecting at O. AD is parallel to BC and AB is parallel to CD since $\square ABCD$ is a parallelogram. Since alternate interior angles formed by parallel line are congruent, $\angle CAD \cong \angle ACB$ and $\angle ADB \cong \angle DBC$. $AD \cong BC$, since opposite sides of a parallelogram are equal in length. Thus, $\triangle AOD \cong \triangle COB$ by Angle-Side-Angle congruence. Thus, $AO \cong OC$ and $BO \cong OD$, since corresponding parts of congruent triangles are congruent.*

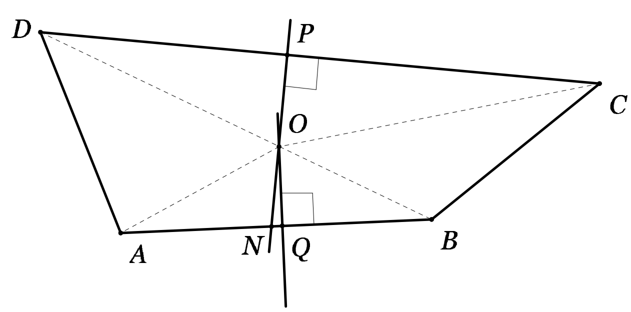
2. Here are two valid proofs of the theorem that the base angles of an isosceles triangle are equal. Which do you like better, and why?

   Proof 1: Given isosceles triangle $\triangle ABC$ with $AC \cong BC$. Construct the bisector of $\angle C$. Let D be the intersection of AB and this bisector. Then $\angle ACD \cong \angle BCD$ and $CD \cong CD$. Therefore $\triangle ACD \cong \triangle BCD$ by SAS congruence. Therefore, $\angle A \cong \angle B$.

   Proof 2: Given isosceles triangle $\triangle ABC$ with $AC \cong BC$. Then $BC \cong AC$ and $\angle ACB \cong \angle BCA$. Thus the parts AC, BC, and $\angle ACB$ of $\triangle ACB$ are congruent, respectively, to the corresponding parts BC, AC and $\angle BCA$ of $\triangle BCA$. Therefore, $\triangle ACB \cong \triangle BCA$ by SAS congruence. Therefore $\angle BAC \cong \angle ABC$.

3. Discover the fallacy in the following "proof": *If two opposite sides of a quadrilateral are congruent, then the remaining two sides must be parallel.*

   Proof: In quadrilateral $\square ABCD$, let $AD = BC$. Assume that AB and DC are not parallel. Let P and Q be the midpoints of DC and AB,respectively, and construct the perpendicular bisectors at P and Q. Then these two bisectors meet in a point O. Let $\{N\} = \overrightarrow{PO} \cap AB$.

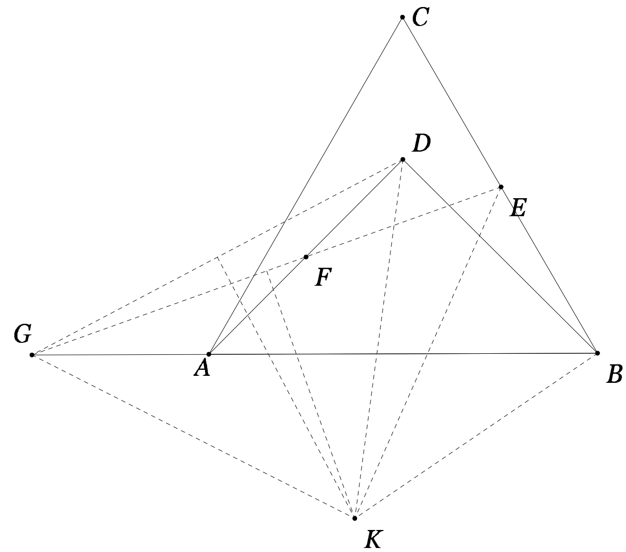


   Since O lies on the perpendicular bisector of CD, $CO \cong DO$. Similarly, $OA \cong OB$. We were given that $AD \cong BC$, so $\triangle ADO \cong \triangle BCO$ by SSS congruence and $\angle AOD \cong \angle BOC$.

We can easily establish that $\angle DOP \cong \angle COP$. By addition, $\angle AOP \cong \angle BOP$. The supplements of these angles must also be congruent: $\angle AON \cong BON$. But, because $\triangle AOQ \cong \triangle BOQ$ by SSS congruence, $\angle AOQ \cong \angle BOQ$. Because the angle bisector is unique, then $\overrightarrow{ON}$ and $\overrightarrow{OQ}$ must coincide and the perpendiculars to these must also be parallel. Hence $AB \parallel CD$.

You can repeat this proof for O outside the circle and for O on DC.

4. Discover the fallacy in the following "proof": $45° = 60°$.



Proof: Construct an equilateral triangle $\triangle ABC$. On side AB construct an isosceles right triangle $\triangle ADB$ with AB as the hypotenuse. Construct EB on BC so that $EB \cong BD$. Let F be the midpoint of AD and connect E to F with a ray that extended will meet $\overleftrightarrow{AB}$ in a point G. Construct GD. Now construct the perpendicular bisectors of GD and GE. Because GD and GE are not parallel, the perpendicular bisectors must meet at a point K. Connect K with points G, D, E, and B.

Now, $GK \cong KD$ and $GK \cong KE$ (K lies on the perpendicular bisector of each of the segments GD adn GE), $KD \cong KE$. By construction $DB \cong EB$. Therefore $\triangle KBD \cong \triangle KBE$ by SSS congruence and $\angle KBD \cong \angle KBE$. By subtraction $\angle DBG \cong \angle EBG$. But $m\angle DBG = 45°$ and $m\angle CBG = 60°$. Therefore, $45° = 60°$.