MATH 330 — Spring 2011
ASSIGNMENT 8

Due March 9, 2011

Recall that for any integer $k \geq 0$ and any real number n we write

$$\binom{n}{k} := \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}.$$

For an integer n with $n \geq k$, these are binomial coefficients, and are entries in Pascal's triangle. For any real value of n, Newton's binomial theorem may then be expressed as

$$(1+x)^n = \sum_{k=0}^{\infty} \binom{n}{k} x^k.$$

Note that for an integer n with $n < k$, $\binom{n}{k} = 0$, so there are only finitely many terms on the right side in the above equation.

8.1. (7 points) Expand $\frac{1}{\sqrt{1+3x}}$ as an infinite series by using the binomial theorem. Either provide your answer in sigma notation or explicitly give a formula for each coefficient in the infinite series (without using binomial coefficient notation).

8.2. (7 points) Using Newton's method for computation of square roots given on page 170 of *Journey Through Genius*, compute $\sqrt{11}$ using the first eight terms of the relevant binomial expansion. To how many digits is this accurate? (You can check this with a calculator.)

8.3. (6 points) Prove that

$$\frac{1}{\sqrt{1-4x}} = \sum_{k=0}^{\infty} \binom{2k}{k} x^k.$$

Hint: You might need to know that

$$\frac{2^n \cdot 1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} = \frac{(2n)!}{(n!)^2}.$$