MATH 330 — Spring 2011
ASSIGNMENT 9

Due March 23, 2011

9.1 (8 points) Recall that the k-th triangular number is $T_k := \binom{k+1}{2} = \frac{(k+1)k}{2}$. On pages 186-187 of *Journey Through Genius*, Dunham presents a proof by Leibniz that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{T_k} = 2$. Translate Leibniz's proof into sigma notation and explain completely each step of the proof.

9.2. (12 points) For $b \neq 0$ and $d > 1$, consider the series

$$\frac{1}{b} + \frac{2}{b \cdot d} + \frac{3}{b \cdot d^2} + \frac{4}{b \cdot d^3} + \frac{5}{b \cdot d^4} + \cdots$$

(a) (2 points) Write this sum using sigma notation.

(b) (5 points) Prove that

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$$

in two different ways: first use the quotient rule; second expand this as a series and use the power rule term by term.

(c) (5 points) Prove that

$$\frac{1}{b} + \frac{2}{b \cdot d} + \frac{3}{b \cdot d^2} + \frac{4}{b \cdot d^3} + \frac{5}{b \cdot d^4} + \cdots = \frac{d^2}{b(d-1)^2}$$

using *only* the term by term power rule and the derivative formula in part (2).

NOTE: The sum of the series in Problem 2 was originally included by Jakob Bernoulli in his *Tractatus de seriebus infinitis*. His method of proof was similar to that used to prove divergence of the harmonic series on pages 196-198 of *Journey Through Genius*.