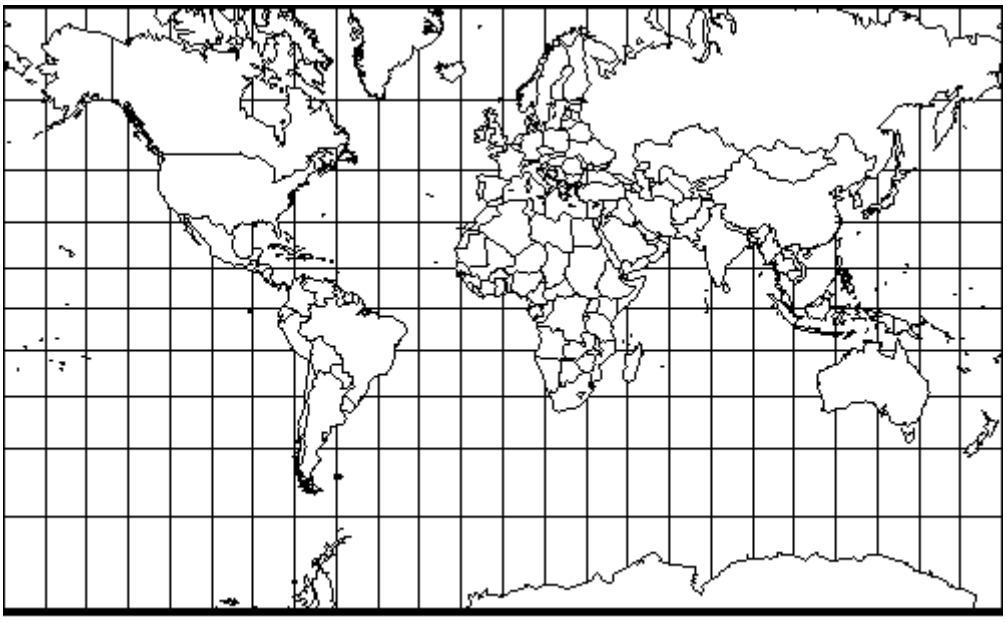
# Mathematics and Maps



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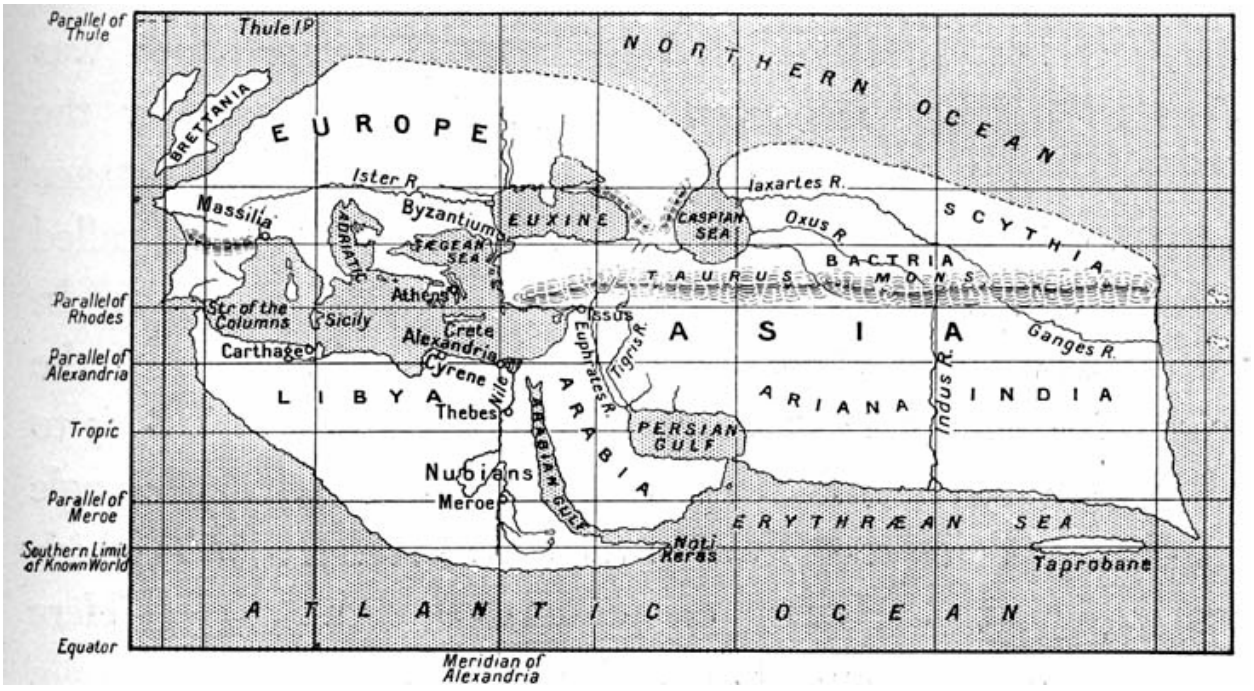
# History

Early world maps reflect the religious beliefs of the form of the world. For example maps have been discovered on Babylonian clay tablets dating from around 600 B.C. One such map shows Babylon and the surrounding area in a stylized form with Babylon represented by a rectangle and the Euphrates river by vertical lines. The area shown is depicted as circular surrounded by water which fits the religious image of the world in which the Babylonians believed.

Eratosthenes, around 250 B.C., made major contributions to cartography. He measured the circumference of the Earth with great accuracy. He sketched, quite precisely, the route of the Nile to Khartoum, showing the two Ethiopian tributaries. He made another important contribution in using a grid to locate positions of places on the Earth. He was not the first to use such a grid for Dicaearchus, a follower of Aristotle, had devised one about 50 years earlier. Today we use latitude and longitude to determine such coordinates and Eratosthenes' grid was of a similar nature. Note, of course, that the use of such positional grids are an early form of Cartesian geometry. Following Dicaearchus, Eratosthenes chose a line through Rhodes and the Pillars of Hercules (present day Gibraltar) to form one of the principal lines of his grid. This line is, to a quite high degree of accuracy, 36° north and Eratosthenes chose it since it divided the world as he knew it into two fairly equal parts and defined the longest east-west extent known. He chose a defining line for the north-south lines of his grid through Rhodes and drew seven parallel lines to each of his defining lines to form a rectangular grid.



Hipparchus was critical of the grid defined by Eratosthenes, saying reasonably enough that it was chosen arbitrarily. He suggested that a grid should be chosen with astronomical significance so that, for example, points on the same line would all have the same length of longest day. Although Hipparchus never constructed a map as far as we know, he did make astronomical observations to describe eleven parallels given by his astronomical definition. Although no copies of the work by Eratosthenes and Hipparchus survives, we know of it through the *Geographical Sketches* of Strabo which was completed in about 23 A.D. Although Strabo gives a critical account of earlier contributions to cartography, he devotes only a small discussion to the problem of projecting a sphere onto a plane. He states clearly that his work is not aimed at mathematicians, rather at statesmen who need to know about the customs of the people and the natural resources of the land.

The final ancient Greek contribution we consider was the most important and, unlike that of Strabo, was written by a noted mathematician. In about 140 A.D. Ptolemy wrote his major work *Guide to Geography,* in eight books, which attempted to map the known world giving coordinates of the major places in terms of what are essentially latitude and longitude. The first volume gives the basic principles of cartography and considers the problem of map projection, that is mapping the sphere onto the plane. He gave two examples of projections, and also described the construction of globes. Right at the beginning Ptolemy identifies two distinct types of cartography, the first being:-

*... an imitation through drawing of the entire known part of the world together with the things which are, broadly speaking, connected with it.*

The second type is:-

*... an independent discipline* [*which*] *sets out the individual localities.*

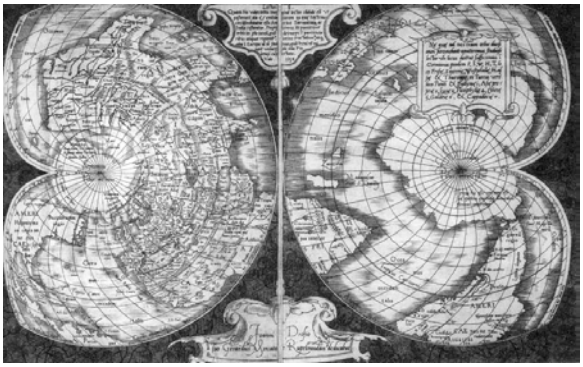
Now the main part of *Geography* consisted of maps but Ptolemy knew that although a scribe could copy a text fairly accurately, there was little chance that maps could be successfully copied. He therefore ensured that the work contained the data and the information necessary for someone to redraw the maps. He followed previous cartographers in dividing the circle of the equator into 360° and took the equator as the basis for the north-south coordinate system. Thus the line of latitude through Rhodes and the Pillars of Hercules (present day Gibraltar) was 36° and this line divided the world as Ptolemy knew it fairly equally into two. The problem of defining lines of longitude is more difficult. It required the choice of an arbitrary zero but it also required a knowledge of the circumference of the Earth in order to have degrees correspond correctly to distance. Ptolemy chose the Fortune Islands (which we believe are the Canary Islands) as longitude zero since it was the most western point known to him. He then marked off where the lines of longitude crossed the parallel of Rhodes, taking 400 stadia per degree.


*Ptolemy's map of the world*.

Had Ptolemy taken the value of the circumference of the Earth worked out by Eratosthenes then his coordinates would have been very accurate. However he used the later value computed by Posidonius around 100 B.C. which, although computer using the correct mathematical theory, is highly inaccurate. Therefore instead of the Mediterranean covering 42° as it should, it covers 62° in Ptolemy's coordinates. Books 2 to 7 of *Geography* contain the coordinates of about 8000 places, but although he knew the correct mathematical theory to compute such coordinates accurately from astronomical observations, there were only a handful of places for which such information existed. It is not surprising that the maps given by Ptolemy were quite inaccurate in many places for he could not be expected to do more than use the available data and, for anything outside the Roman Empire, this was of very poor quality with even some parts of the Roman Empire severely distorted.

At a time when Christian Europe was producing religious representations of the world rather than scientific maps, another type of map, or perhaps more accurately chart, for the use of sailors began to appear. These were called *portolan* maps (from the Italian word for a sailing manual) and were produced by sailors using a magnetic compass. The earliest examples we know about date from the beginning of the 14th century, and were Italian or Catalan portolan maps. The earliest portolan maps covered the Mediterranean and Black Sea and showed wind directions and such information useful to sailors. The coast lines shown on these maps are by far the most accurate to have been produced up to that time. The Catalan World Map produced in 1375 was the work of Abraham Cresques from Palma in Majorca. He was a skilled cartographer and instrument maker and the map was commissioned by Charles V of France. The western part of his map was partly based on portolan maps while the eastern part was based on Ptolemy's data.

Mercator made many new maps and globes, but his greatest contribution to cartography must be the *Mercator projection*. He realized that sailors incorrectly assumed that following a particular compass course would have them travel in a straight line. A ship sailing towards the same point of the compass would follow a curve called a loxodrome (also called a rhumb line or spherical helix), a curve which Pedro Nunez, a mathematician greatly admired by Mercator, had studied shortly before. A new globe which Gerardus Mercator produced in 1541 was the first to have rhumb lines shown on it. This work was an important stage in his developing the idea of the Mercator projection which he first used in 1569 for a wall map of the world on 18 separate sheets. The 'Mercator projection' has the property that lines of longitude, lines of latitude and rhomb lines all appear as straight lines on the map. In this projection the meridians are vertical and parallels having increased spacing in proportion to the secant of the latitude. Edward Wright published mathematical tables to be used in calculating Mercator's projection in 1599.



Mercator's World Map


A Mercator Map of the Americas

# Latitude and Longitude

## DEFINITIONS:

Latitude

Measurement of distance in degrees north or south of the Equator; from the Latin *latus*, meaning "wide".

Longitude

Measurement of distance in degrees east or west of the prime meridian; from the Latin *longus*, meaning "length".

## WHERE ARE YOU?

The Earth's circumference is divided into 360 degrees (you should keep a world globe handy when introducing these topics), each degree represented by an imaginary line running either east-west or north-south on the surface of the Earth. The prime meridian is the imaginary line which runs through the geographic North Pole to the geographic South Pole, passing through Greenwich England. The prime meridian is defined as 0º E and 0º W. Each meridian (longitude line) to the west of Greenwich is labeled 1-180º W, while each meridian to the east of Greenwich is labeled 1-180º E.

**Why was Greenwich, England selected as 0º E and 0º W?** (Find out on the Internet!)

Since the Earth's circumference (a complete circle) is 360 degrees, then directly opposite the prime meridian is a meridian which can be labeled both as 180º E and 180 ºW. This meridian is called the International Date Line. Likewise, the Equator is 0º N and 0º S latitude. The geographic North Pole is 90º N latitude, while the South Pole is 90º S latitude.

**Why is 180 degE and 180 degW labeled the International Date Line?**

Therefore, if you are trying to describe to anyone exactly where you're located on the planet Earth, latitude and longitude will provide you with a means of answering the question "Where are you?".