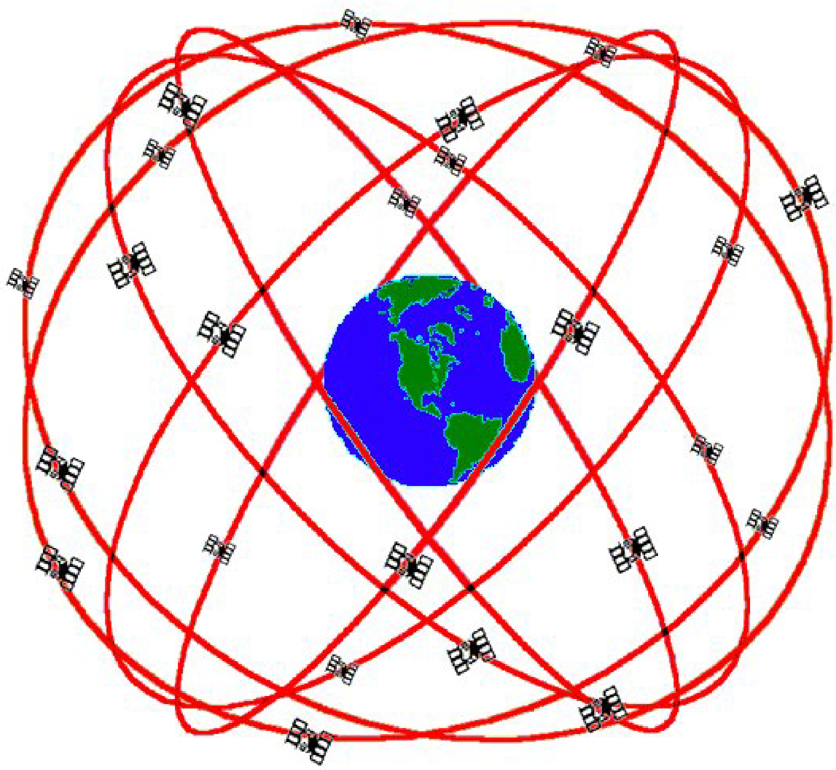
GPS and Mathematics
NCTM 2003



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Some Mathematics

Remember that the speed of light is 186,272 miles per second.

1. A GPS receiver picked up the signals from three different satellites and measured the time-delay for each one. Satellite A took 0.080000 seconds, satellite B took 0.065000 seconds, and satellite C took 0.070000 seconds. What is the distance from the GPS receiver to each satellite? Draw a correctly proportioned sketch of the three satellites, the point of the GPS receiver on earth, and the paths of the radio signals. The radius of the earth is 3,963 miles.

2. (a) If the satellite and the GPS receiver are out of sync by $1/100^{th}$ of a second, how far off could the distance measurement be?
   (b) A clock accuracy of one billionth of a second per ear allows an error of one second in approximately how many years? Show the steps and units for your calculation.

3. If a satellite is 11,176 miles from a receiver, how long does it take for the signal to travel from the satellite to the receiver?

4. Look up the terms latitude and longitude on the Internet. Find a location that will give you the longitude, latitude, and the altitude for each of the following:

   a. Washington, DC;
   b. Denver, CO;
   c. Pikes Peak in Colorado;
   d. Mt. Shasta in California;
   e. the South Pole;
   f. Sydney, Australia;
   g. Greenwich, England;
   h. Diego Garcia in the Indian Ocean;
   i. Wrangle Island in the Artic;
   j. your home town.

5. Plot and label the following points on an x, y, z coordinate system:
   (1,0,0); (2,0,0); (0,1,0); (0,0,1); (1,1,0); (1,0,1); (0,1,1) and (3,3,3)

6. Draw an x, y, z coordinate system and show how to use the Pythagorean Theorem to calculate the following distances:

   a. distance between (1,2,0) and (0,0,0);
   b. distance between (0,1,2) and (0,0,0); and
   c. distance between (1,2,1) and (3,3,3).

7. Since a mariner traveling at sea is at sea level, only two satellites are required to determine a fix. This is true because the ship is at the intersection of three spheres, one of which is the sphere of the earth. Solve the problem of calculating the fix in x, y, z coordinates by putting the origin of the coordinate system at the center of the earth with radius r_c. Run the y-axis through satellite 1 at $(0,k,0)$ at a distance of r_1, and put satellite 2 in the zy-plane at $(0,a,b)$ at distance r_2.

   a. Give the equation of the three spheres.
   b. Eliminate z and x and solve for y.
   c. Eliminate x and solve for z in terms of y.

d. Solve for x in terms of y and z.

e. Draw an x,y,z coordinate system showing the receiver at $P(x,y,z)$ in the first octant, and the two satellites and three spheres.

8. In the work done with three satellites we put the center of our coordinate system at satellite 1, then put the y-axis through satellite 2, and then put satellite 3 in the xy-plane determined by satellites 1 and 2. In this case we had:

$$x^2 + y^2 + z^2 = r_1^2$$
$$x^2 + (y-k)^2 + z^2 = r_2^2$$
$$(x-a)^2 + (y-b)^2 + z^2 = r_3^2$$

from which we got

$$y = \frac{r_2^2 - r_1^2 - k^2}{-2k}$$
$$x = \frac{r_3^2 - r_1^2 - b^2 - a^2 + 2by}{-2a}$$
$$z = \pm\sqrt{r_1^2 - x^2 - y^2}$$

Now, you are given that $r_1 = r_2 = r_3 = 12,180$ and based on the locations of the satellites we find that $k = 22,402.77$, $a = 9,533.55$ and $b = 4,605.53$. Find the coordinates of the points at which the three spheres intersect. Now, since we changed the origin of our coordinate system from the center of the earth to satellite 1, we have to find the sphere that defines the surface of the earth. This is done by solving a system of three equations in three unknowns. Doing this we find that the earth is now a sphere centered at the point (–3716.73,–10542.41, 1443.23) with radius 3963. Can you use this information to eliminate one of the two points you found above?

Mathematical View

Another way to understand the operation of a GPS system is to look at the math that goes into calculating a position. From Pythagoras we have:

$$Prs + T + Es = \sqrt{(x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2}$$

where x, y, z are the positions we are trying to find and T is the time error at the receiver. The terms x_s, y_s, z_s are the satellite positions that can be calculated from ephemeris information sent from each satellite. The Es term is a sum of all the modeling errors considered by the GPS. These include such things as tropospheric and ionospheric errors, clock errors from the satellite and any other error the GPS receivers thinks is significant enough to model. Prs is the approximate (pseudo-range) distance from the receiver to the satellite. We can calculate the pseudo-range and satellite positions independently and we can factor in modeling information from hard-coded data. Thus we are left with four unknowns, x, y, z, and T. Therefore we need 4 equations to solve for the 4 unknowns. Mathematically this is a standard least squares problem.

In actual practice a ***Garmin*** receiver calculates a set of equations with 7 unknowns. In addition to the 3 positions and time they have added the Doppler data *dx*, *dy*, and *dz* which represents the relative speed between the satellite and the receiver. These terms are needed because our solution is based on moving objects and *dx* and *dy* can be used as part of the receiver speed calculation. Four equations will compute a full 3D solution but new 12 channel Garmin units can use additional satellites to perform an over-determined solution which will offer more accuracy.

The Other Two Elements of the GPS System

In addition to the receiver we must have a set of satellites in the sky and a method of updating the data in each satellite. There are full time land based sites that monitor the various satellites which are often referred to as Space Vehicles, SV's. These land based sites check the health of the SV's, check how close they are to their optimum orbits, check the clock accuracy, and send adjustments as needed. The land based sites are located a precisely known positions so that they can verify the operation of the satellites.

The satellites are traveling around the world 11,000 nautical miles high in carefully controlled orbits at a speed that means they will make a complete orbit twice a day. Each orbit takes 11 hours and 58 minutes, so like the stars they will seem to drift 4 minutes a day. The complete constellation consists of a minimum of 21 SV's and 3 working spares. Currently there are 27 total satellites in the sky and it is possible that there could be as many as 31 or 32. There are 6 orbits with multiple satellites in each orbit as depicted in the drawing at the top of the page. Each orbit is inclined 55 degree from the equator and thus there are no orbits that go directly over the poles, but certainly a great many orbits can be seen from the poles or anywhere else on the earth. The goal of the system is to always provide at least 4 satellites somewhere in the visible sky. In practice there are usually many more than this; sometimes as many as 12.

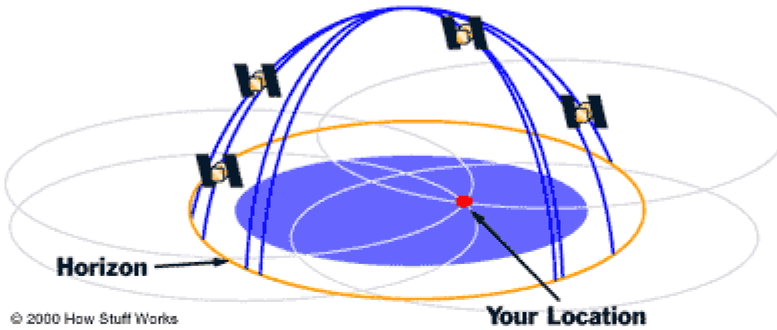
Each satellite contains a supply of fuel and small servo engines so that it can be moved in orbit to correct for positioning errors. With update control from the ground units it can maintain an essentially circular orbit around the earth. It also contains a receiver to get update information, a transmitter to send information to the GPS receiver, an antenna array to magnify the weak transmitter signal, several atomic clocks to accurately know the time, control hardware, and photoelectric cells to power everything.

How GPS Receivers Work[1]

How It Works

The **Global Positioning System** (GPS) consists of 24 earth-orbiting satellites. These satellites allow any person who owns a **GPS receiver** to determine his or her precise longitude, latitude and altitude anywhere on the planet. For as little as $100, you can know exactly where you are and where you have been. For anyone who has ever been lost — while hiking in the woods, boating in the ocean, driving in a unfamiliar city or flying a small airplane at night — a GPS receiver is a miracle. When you use GPS receiver, you're never lost![2]

[1] Based on information from the website "How Things Work" by Marshall Brain & Tom Harris
[2] If you don't have a map, GPS information may not be much help. One can be lost even with the GPS!!

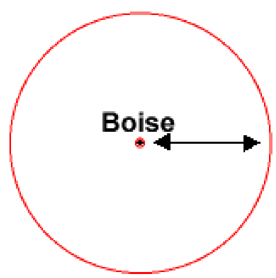


How is this possible? We will look at the details of how the GPS satellites and GPS receivers work together to pinpoint a location.

Trilateration[3]

In order to understand how the GPS satellite system works, it is very helpful to understand the concept of **trilateration**. Let's look at an example to see how trilateration works.

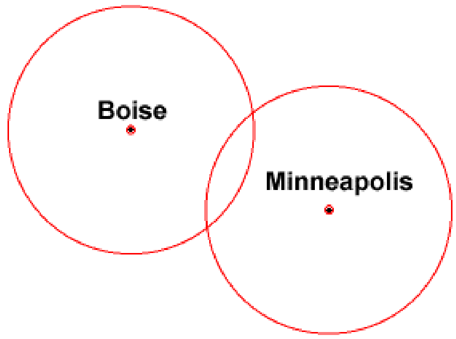
Let's say that you are somewhere in the United States and you are TOTALLY lost — you don't have a clue where you are. You find a friendly-looking person and ask, "Where am I?" and the person says to you, "You are 625 miles from Boise, Idaho." This is a piece of information, but it is not really that useful by itself. You could be anywhere on a circle around Boise that has a radius of 625 miles, like this:



If you know you are 625 miles from Boise,
you could be anywhere on this circle.

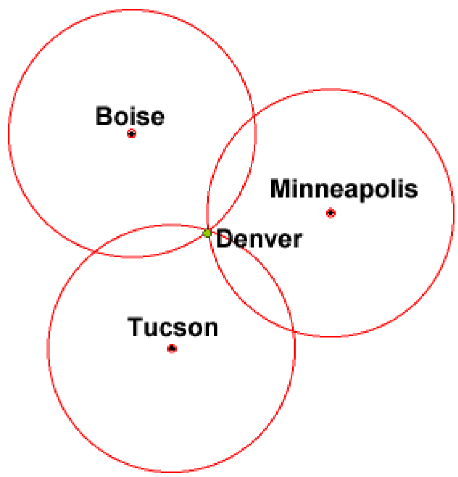
So you ask another person, and he says, "You are 690 miles away from Minneapolis, Minnesota." This is helpful — if you combine this information with the Boise information, you have two circles that intersect. You now know that you are at one of two points, but you don't know which one, like this:

[3] Trilateration as opposed to triangulation.



If you know you are 625 miles from Boise and 690 miles from Minneapolis, then you know you must be at one of two points.

If a third person tells you that you are 615 miles from Tucson, Arizona, you can figure out which of the two points you are at:



With three known points, you can determine that your exact location is somewhere near Denver, Colorado!

With three known points, you can see that you are near Denver, Colorado!

Trilateration is a basic geometric principle that allows you to find one location if you know its distance from other, already known locations. The geometry behind this is very easy to understand in two-dimensional space.

This same concept works in three-dimensional space as well, but you're dealing with spheres instead of circles. You also need 4 spheres instead of three circles to find your exact location. The heart of a GPS receiver is the ability to find the receiver's distance from 4 (or more) GPS satellites. Once it determines its distance from the four satellites, the receiver can calculate its exact location and altitude on Earth! If the receiver can only find three satellites, then it can use an imaginary sphere to represent the earth and can give you location information but no altitude information.

For a GPS receiver to find your location, it has to determine two things:

The location of at least three satellites above you

The distance between you and each of those satellites

Measuring Distance

GPS satellites send out radio signals that your GPS receiver can detect. But how does the signal let the receiver know how far away the satellite is? The simple answer is: A GPS receiver measures the amount of time it takes for the signal to travel from the satellite to the receiver. Since we know how fast radio signals travel — they are electromagnetic waves and so (in a vacuum) travel at the speed of light, about 186,000 miles per second — we can figure out how far they've traveled by figuring out how long it took for them to arrive.

Measuring the time would be easy if you knew exactly what time the signal left the satellite and exactly what time it arrived at your receiver, and solving this problem is key to the Global Positioning System. One way to solve the problem would be to put extremely accurate and synchronized clocks in the satellites and the receivers. The satellite begins transmitting a long digital pattern, called a **pseudo-random code**, as part of its signal at a certain time, let's say midnight. The receiver begins running the same digital pattern, also exactly at midnight. When the satellite's signal reaches the receiver, its transmission of the pattern will lag a bit behind the receiver's playing of the pattern. The length of the delay is equal to the time of the signal's travel. The receiver multiplies this time by the speed of light to determine how far the signal traveled. If the signal traveled in a straight line, this distance would be the distance to the satellite.

The only way to implement a system like this would require a level of accuracy only found in atomic clocks. This is because the time measured in these calculations amounts to nanoseconds. To make a GPS using only synchronized clocks, you would need to have **atomic clocks** not only on all the satellites, but also in the receiver itself. Atomic clocks usually cost somewhere between $50,000 and $100,000, which makes them a little too expensive for everyday consumer use!

The Global Positioning System has a very effective solution to this problem — a GPS receiver contains no atomic clock at all. It has a normal quartz clock. The receiver looks at all the signals it is receiving and uses calculations to find both the exact time and the exact location simultaneously. When you measure the distance to four located satellites, you can draw four spheres that all intersect at one point, as illustrated above. Four spheres of this sort will not intersect at one point if you've measured incorrectly. Since the receiver makes all of its time measurements, and therefore its distance measurements, using the clock it is equipped with, the distances will all be proportionally incorrect. The receiver can therefore easily calculate exactly what distance adjustment will cause the four spheres to intersect at one point. This allows it to adjust its clock to adjust its measure of distance. For this reason, a GPS receiver actually keeps extremely accurate time, on the order of the actual atomic clocks in the satellites!

One problem with this method is the measure of speed. As we saw earlier, electromagnetic signals travel through a vacuum at the speed of light. The earth, of course, is not a vacuum, and its atmosphere slows the transmission of the signal according to the particular conditions at that atmospheric location, the angle at which the signal enters it, and so on. A GPS receiver guesses the actual speed of the signal using complex mathematical models of a wide range of atmospheric conditions. The satellites can also transmit additional information to the receiver.

From this discussion you have learned several important facts about the Global Positioning System:

> The Global Positioning System needs 24 satellites so it can guarantee that there are at least 4 of them above the horizon for any point on earth at any time. In general there are normally 8 or so satellites *visible* to a GPS receiver at any given moment.

Each satellite contains an atomic clock[4].
The satellites send radio signals to GPS receivers so that the receivers can find out how far away each satellite is. Because the satellites are orbiting at a distance of 11,000 miles overhead, the signals are fairly weak by the time they reach your receiver. That means you have to be outside in a fairly open area for your GPS receiver to work.

Finding the Satellites

The other crucial component of GPS calculations is the knowledge of where the satellites are. This isn't difficult because the satellites travel in a very high and predictable orbit. The satellites are far enough from the Earth (11,000 miles) that they are not affected by our atmosphere. The GPS receiver simply stores an **almanac** that tells it where every satellite should be at any given time. Things like the pull of the moon and the sun do change the satellites' orbits very slightly, but the Department of Defense constantly monitors their exact positions and transmits any adjustments to all GPS receivers as part of the satellites' signals.

Finding Location

The most essential function of a GPS receiver is to pick up the transmissions of at least four satellites and combine the information in those transmissions with information in an electronic almanac, so that it can mathematically determine the receiver's position on Earth. The basic information a receiver provides, then, is the latitude, longitude and altitude (or some similar measurement) of its current position. Most receivers then combine this data with other information, such as maps, to make the receiver more user-friendly. You can use maps stored in the receiver's memory, connect the receiver up to a computer that can hold more detailed maps in its memory or simply buy a detailed map of your area and find your way using the receiver's latitude and longitude readouts.

Geographers have mapped every corner of the Earth, so you can certainly find maps with the level of detail you desire. You can look at a GPS receiver as an extremely accurate way to get raw positional data, which can then be applied to geographic information that has been accumulated over the years. GPS receivers are an excellent navigation tool, with seemingly endless applications!

What It Can Do

The Global Positioning System, a collection of 24 earth-orbiting satellites, has a number of possible applications, spanning across several areas of society. For most of us, the way we can take advantage of GPS is to purchase a handheld GPS receiver, or have one installed in our car. Handheld receivers are compact, and the most basic ones are fairly affordable. You can pick one up for as low as $100!

This could certainly be a sensible purchase, when you consider all of the things a GPS receiver can do for you. The basic function of a GPS receiver is to figure out its location on Earth. To everyone who's ever lost their way in the woods, driven off course on a cross-country trip or gotten turned around while piloting a boat or airplane, the advantages of this simple function are obvious. But most GPS receivers go far beyond providing this simple navigational data. They can act as an interactive map, and they have a number of recreational applications.

The Basics

At its heart, a GPS receiver is simply a device that can locate itself on Earth. It does this by communicating with at least four satellites overhead. For this reason, a GPS receiver is limited as to where it can function. It has to be able to "see" the satellites to calculate latitude and longitude, which means it usually won't work inside. So, one of the basic characteristics of GPS receivers is that they find your location only when you are outside.

[4] Each satellite contains four atomic clocks — redundancy, you know.

The simplest GPS receiver would give you just the **coordinates** of your location on Earth in latitude, longitude and altitude. Latitude and longitude are basically X and Y axes of a big imaginary grid wrapped around the planet, and altitude is a measure of your distance above sea level. If you had a GPS receiver that gave you these simple coordinates, and you had a map of your area that used this same coordinate system, you could find your location simply by reading the map. In this regard alone, a GPS receiver is a amazing device. Without a GPS receiver, you would have to find your position based on the position of the stars in the sky, using complicated tools and calculations. And you wouldn't have near the same level of accuracy!

But today's handheld GPS receivers give you much more than this raw data. Even low-end receivers have some sort of electronic map stored in memory, so you don't have to carry around a bunch of paper maps. The receiver takes the coordinate information and applies it to its electronic map, graphically pointing out to you where you are in relation to roads, bodies of water, etc. Maps vary a great deal in the level of detail they offer; but the basic idea behind this function is to give you a map that automatically marks your location, without you having to consider your coordinates. This is a great convenience any time you need to use a map, and is extremely helpful at times when you can't take the time to find your location on a map, such as when you're driving down the highway.

GPS in Motion

A standard GPS receiver will not only place you on a map at any particular location, but will also trace your path across a map as you move. If you leave your receiver on, it can stay in constant communication with GPS satellites to see how your location is changing. A receiver must know the exact time to find its location. If it combines these two pieces of information — your changing location and the exact time — it can also calculate how fast you are going. A receiver can use all of this basic data to give you several pieces of valuable information:

- How far you've traveled (odometer)
- How long you've been traveling
- Your current speed (speedometer)
- Your average speed
- A "breadcrumb" trail showing you exactly where you have traveled on the map
- The estimated time of arrival at your destination if you maintained your current speed

To obtain this last piece of information, you would have to have given the receiver the coordinates of your destination, which brings us to another GPS receiver capability: inputting location data.

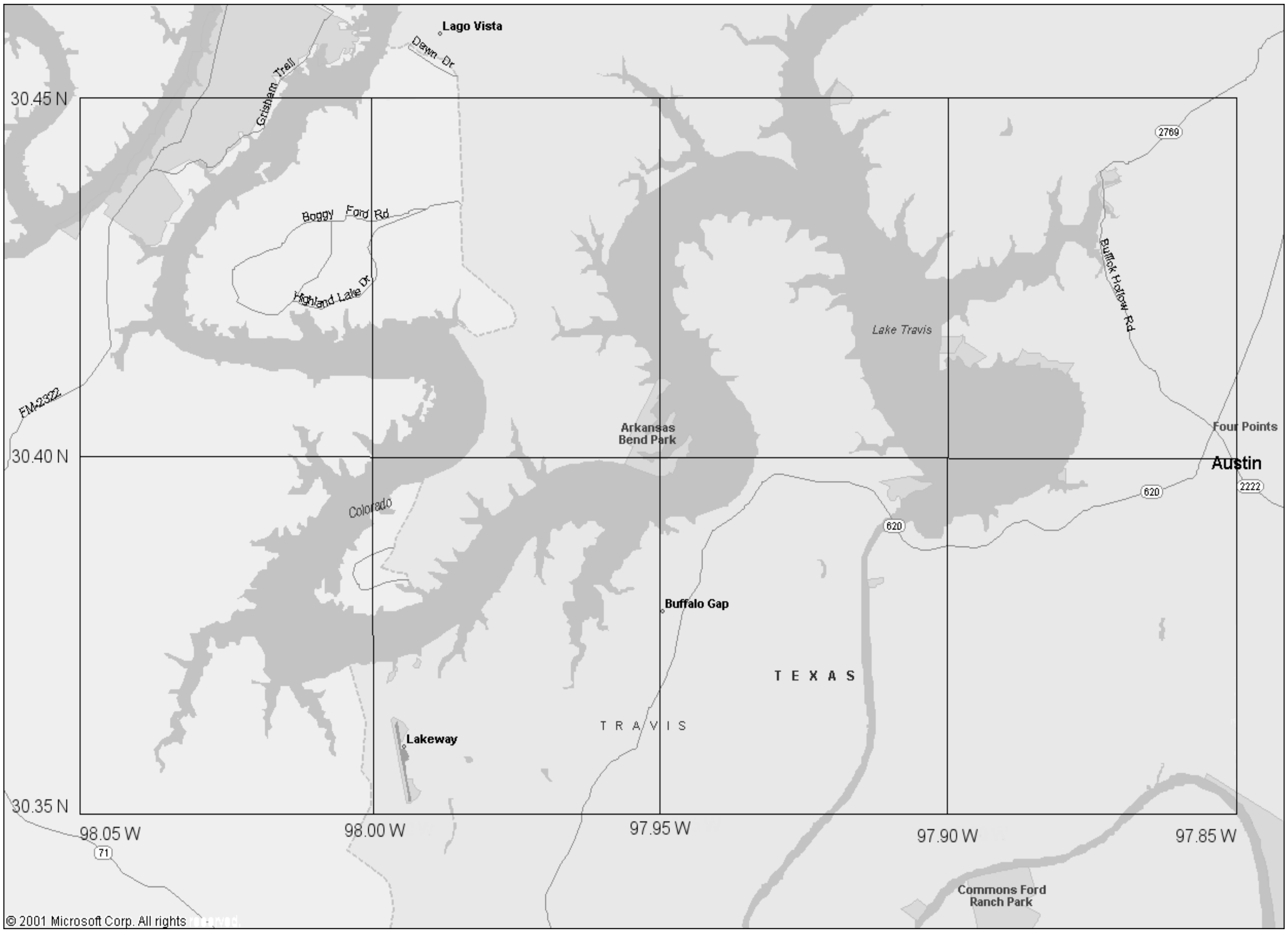
Cool Facts

- The first GPS satellite was launched in 1978.
- The current system is composed of second generation GPS satellites, called Block II.
- The first Block II satellite was launched in 1989.
- The Defense Department declared GPS fully operational in 1995.
- When the system was first introduced, miscalculations were programmed into GPS transmissions to limit the accuracy of non-military GPS receivers. This operation was cancelled in May 2000.
- There are 24 GPS satellites in orbit at this moment.
- The 24 satellites cost an estimated $12 billion to build and launch.
- Each satellite weighs about 1,735 lb (787 kg).
- The satellites are in orbit about 12,500 mi (20,000 km) above the Earth.
- A satellite takes 11 hours 58 minutes to orbit the Earth once.
- The Russians have a system identical to the U.S. system called GLONASS. It is not yet fully operational.

GPS Application 01[5]

Lake Travis, Hill Country, TX

Below is a map of a portion of Lake Travis, together with six screens from a hand-held GPS unit that show longitude and latitude for a fisherman's day on the lake. Use the GPS readings to mark the path and the direction of the trip on the map.



LAT N	30° 26.274′
LON W	97° 52.510′
date	08 APR 03

LAT N	30° 27.588′
LON W	97° 56.211′
date	08 APR 03

LAT N	30° 23.998′
LON W	98° 00.806′
date	08 APR 03

LAT N	30° 24.226′
LON W	97° 58.501′
date	08 APR 03

LAT N	30° 27.196′
LON W	97° 57.398′
date	08 APR 03

LAT N	30° 24.044′
LON W	97° 54.914′
date	08 APR 03

[5] Modeled on GPS Application, pg 61, *Mission Mathematics: Grades 9-12*, NCTM, 1997

GPS Application 02[6]

Using a GPS Handheld Receiver to Determine Distance and Area of Polygonal Plot

Activity Description

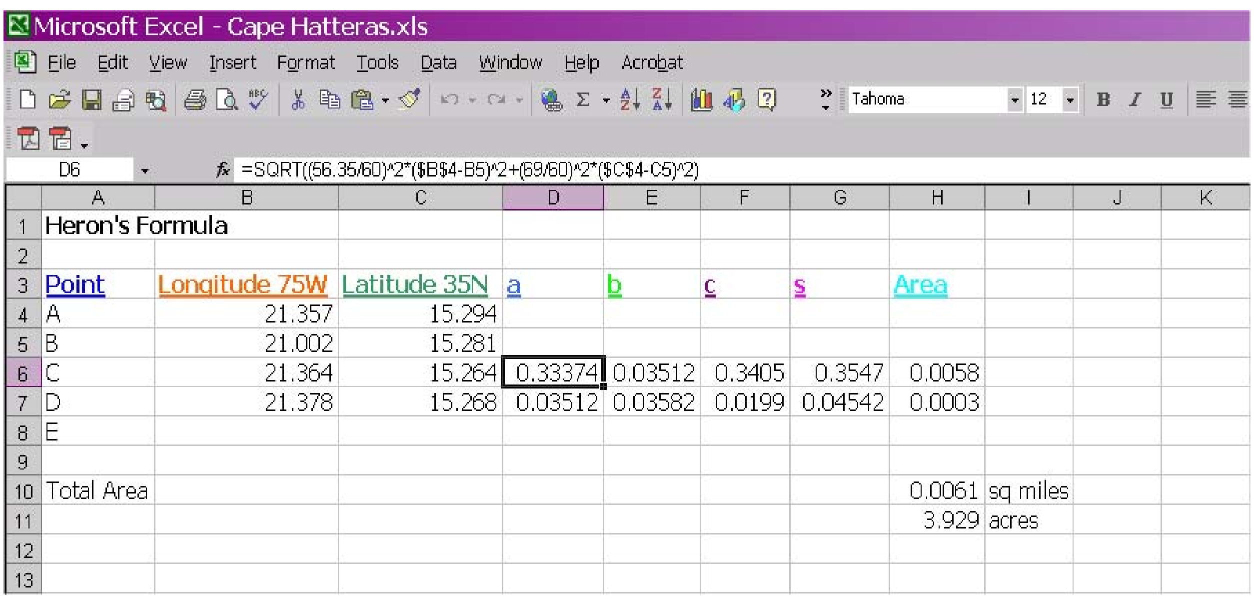
This activity explores how distances and areas of convex polygonal plots of land (and concave plots with modification) can be determined using a GPS, The Geometer's Sketchpad, and Excel.

Mathematics: This activity involves *measurement of longitude and latitude*, the *distance formula*, *Heron's formula,* and *areas of convex* and *concave polygons*.

Mathematical Thinking: *Conversion* of differences in longitude and latitude to miles*, subdividing regions* to determine areas, *devising* a spreadsheet-based general method to calculate areas of polygons.

Technology: The activity utilizes the *position features* of a Global Positioning System, basic *construction* and *measurement features* of the Geometer's Sketchpad, and a basic Excel *spreadsheet*.

Sample Screenshot: A spreadsheet using Heron's formula to calculate the area of the rectangular lot, subdivided into two triangular regions, at *Cape Hatteras NC*.



	A	B	C	D	E	F	G	H	I	J	K
1	Heron's Formula										
2											
3	Point	Longitude 75W	Latitude 35N	a	b	c	s	Area			
4	A	21.357	15.294								
5	B	21.002	15.281								
6	C	21.364	15.264	0.33374	0.03512	0.3405	0.3547	0.0058			
7	D	21.378	15.268	0.03512	0.03582	0.0199	0.04542	0.0003			
8	E										
9											
10	Total Area							0.0061	sq miles		
11								3.929	acres		
12											
13											

[6] Center for Technology and Teacher Education, University of Virginia URL: http://www.teacherlink.org/content/math/activities/gps.html

Activity Guide

Part 1: Determining longitude and latitude of a plot of land

(1) Define the terms *polygon* and *regular polygon*.

(2) Identify a nearby plot of land that has a polygonal shape and accessible vertices (a field at your school, your house lot, etc.). Roughly approximate the longitude and latitude of this plot of land. Discuss how you made these approximations.

(3) Use a GPS to determine the longitude and latitude of each vertex of the plot. Record your coordinates.

Example: Below are the coordinates of the new lot for the Cape Hatteras lighthouse in Buxton NC. These coordinates are used in the spreadsheet screenshots on the cover page.

Vertex A W 75 21.357 'N 35 15.294'
Vertex B W 75 21.002' N 35 15.281'
Vertex C W 75 21.364' N 35 15.264'
Vertex D W 75 21.378' N 35 15.268'

(4) Compare your approximations to your GPS readings.

Part 2: Converting differences in longitude and latitude to linear distances

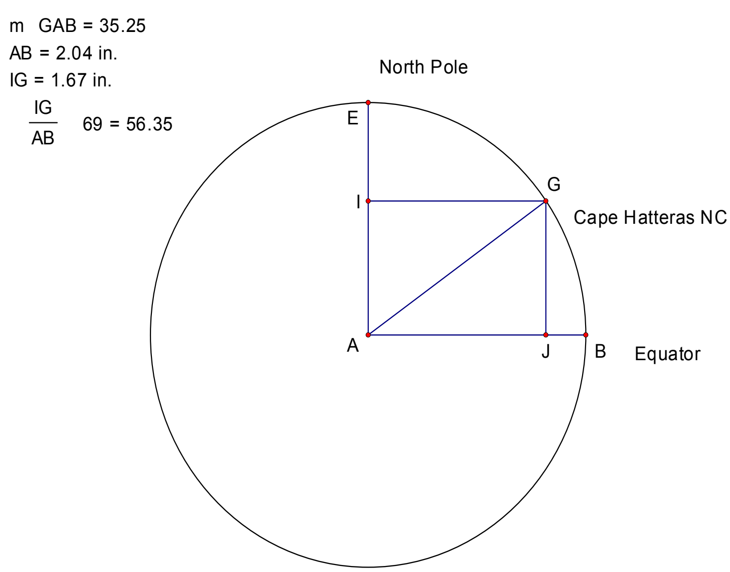
(5) Draw a sketch of your plot of land and label the vertices in clockwise order. Discuss appropriate units of measurement for the lengths of the sides and the area of this plot. Estimate the lengths of the sides of the plot.

(6) Discuss how you would you convert differences in latitude to linear distances and how you would convert differences in longitude to linear distances. How would you calculate the linear distances between vertices of your plot?

(7) Determine the number of miles per degree of latitude and the number of miles per degree of longitude at the location of your plot.

Solution 1: Recall that lines of latitude are approximately 69 miles apart. Hence, there are 69 miles per degree of latitude or 69/60 miles per second of latitude.

Solution 2: Note that lines of longitude are approximately 69 miles apart at the equator but meet at the poles. Hence, there are 69 miles per degree or 69/60 miles per second of longitude at the equator, but 0 miles at the poles. One way to determine the number of miles per degree of longitude at your latitude is as follows: Draw a sketch of the Earth using the Geometer's Sketchpad. Place points to mark the equator, the poles, and your location by latitude. The figure below shows Cape Hatteras at N 35.25 latitude.



At any latitude, the number of miles per degree of longitude equals the circumference of the Earth at that latitude divided by 360. The ratio of the circumference around the Earth at your location (e.g., G) to the circumference at the equator (B) is equal to the ratio of the radii (i.e., segment IG and segment AB) at those latitudes (why?). Set up the ratio between the radius of the Earth at your latitude and that at the equator. Since there are 69 miles per degree of longitude at the equator, this ratio multiplied by 69 will equal the number of miles per degree of longitude at your location.

Note: By moving *point G* on this sketch you can determine the number of miles per degree of longitude at any latitude.

(8) Calculate the lengths of the sides of your plot and record them on the sketch of your plot. Calculate the perimeter of your plot.

Hint: Use the number of miles per degree of latitude and the number of miles per degree of longitude (at your latitude) as "conversion factors" in the distance formula. The formula in the spreadsheet on the activity description page uses a conversion factor of 69/60 miles for each second of latitude and 56.35/60 for each second of longitude at N 35.25 latitude to calculate the lengths of the sides of the lot at the Cape Hatteras Lighthouse.

Part 3: Determining areas of polygons with Heron's Formula

(9) Discuss how to find the area of various polygons, including those that are not regular. How would you find the area of an n-sided polygon?

(10) Estimate the area of your plot. Discuss how you would calculate the area of the plot.

(11) What is Heron's formula?

Heron's Formula: Heron's Formula relates the area of a triangle to the lengths of its sides. If a, b, and c are the lengths of the sides of a triangle, then the area of the triangle is $\sqrt{s(s-a)(s-b)(s-c)}$ where *s*, the semi-perimeter, is equal to

$$s = \frac{a+b+c}{2}.$$

(12) Use Heron's formula to calculate the area of your plot.

Note: Subdividing your plot into triangles and applying Heron's formula to each triangular subdivision will yield the area.

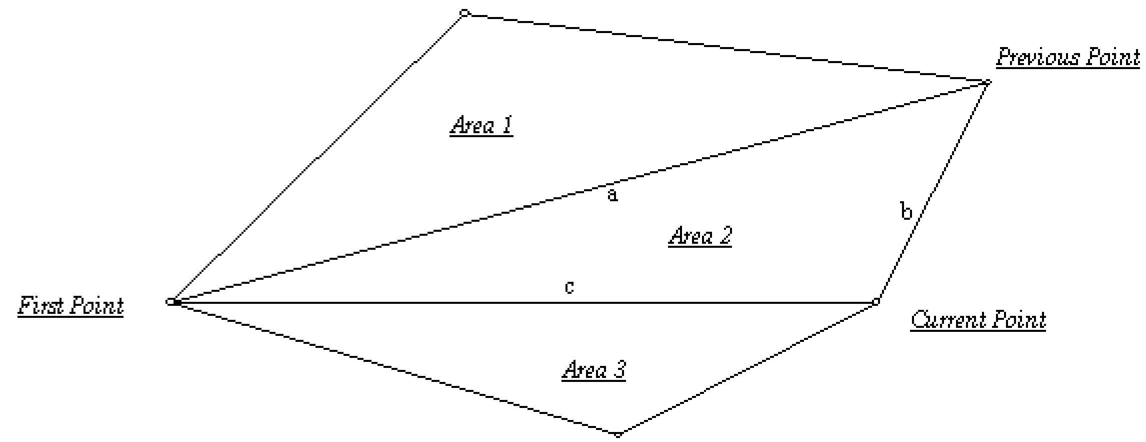
Part 4: Generalizing the method to any convex polygonal plot of land

(13) Discuss how to generalize the method used to calculate the area of your plot to any convex polygon.

(14) Create a spreadsheet where the input is a set of coordinates for the vertices of any convex polygon and the output is the area of the polygon.

A Solution: The spreadsheet above shows the coordinates and area calculation for the lot at Cape Hatteras. The lot is a quadrilateral and is partitioned into two triangles. With vertices input in clockwise order, the spreadsheet first calculates the lengths of sides *a*, *b*, and *c* and the semi-perimeter *s* for each triangle. It then uses Heron's formula to calculate the areas of each of the two triangles that partition this convex polygonal area and then sums these areas to calculate the total area. Each triangle uses a common "first point" as a vertex, along with a "current point" and a "previous point." See the Sketchpad sketch and solution below for a description of the spreadsheet as applied to a convex polygonal region subdivided into three triangles.

Solution: The spreadsheet formula partitions the area into triangles, all originating from the first vertex. Going clockwise, *a* is calculated as the distance between the first and previous vertices, *b* as the distance between the previous and the current vertex, and *c* as the distance between the current vertex and the first vertex. Entering the formula in the spreadsheet this way allows us to fill down as more vertices are added.



Note 1: In the spreadsheet, data were entered as minutes, in decimal form. GPS units allow one to choose different data formats. The degree, minute, second format requires that data be entered in the spreadsheet in the **[h]:mm:ss** cell format, and then copied to cell with a general format. This will display the readings as days and when multiplied by 24 gives hours in decimal form. Hence it is easier to record GPS data in a form ready for use in calculations.

Note 2: This calculation does not account for the curvature of the Earth. For small distances the curvature can be ignored.

Note 3: There are 640 acres in a square mile.

(15) Identify one or two other accessible *convex* plots of land. Estimate their areas. Use your GPS and spreadsheet to calculate their areas.

Extension:

(16) Identify an accessible *concave* plot of land. Estimate the area of this plot. Discuss how you can use the GPS and your spreadsheet to calculate the area of this plot. Calculate the area.

Hint: If your convex polygon has one "concave" vertex, the spreadsheet described above will work if the concave vertex is chosen as the "first point" common to all triangular subdivisions.

Resources

References:

Chew, L. (1998). GPS I: Fundamentals global positioning systems. Paper presented at the meeting of AAPT/PTRA Workshop, Orlando, FL.

Garmin Corporation. (1999). GPS 12 personal navigator owner's manual and reference. p. 6-7.

Kaplan, E. D. (1996). Understanding GPS principles and applications. Boston: Arte House.

Leick, A. (1995). GPS satellite surveying. New York: John Wiley and Sons.

Parkinson, B. W. (1996). Introduction and heritage of NAVSTAR, the global positioning system. In B. W. Parkinson & J. J. Spilker, Jr. (Eds.), Global positioning system: Theory and applications (pp. 3-28). Washington, DC: American Institute of Aeronautics and Astronautics, Inc.

Related Websites:

GPS: A New Constellation http://www.nasm.edu/gps/intro.html
This website describes navigation before the invention of the GPS, how the GPS has revolutionized, how the GPS works, and presents other navigational information.

GPS: Global Positioning System Overview
http://www.colorado.edu/geography/gcraft/notes/gps/gps_f.html
This site provides a good overview of the GPS.

GPS World http://www.gpsworld.com/
This website provides a 2000 GPS Buyers Guide, GPS articles, and other various information.

How Stuff Works http://www.howstuffworks.com/gps.htm
This is a good starting point for finding more information about how the GPS works.

Look-up Latitude and Longitude http://www.bcca.org/misc/qiblih/latlong.html
At this site you can search for the latitude and longitude of various major cities in the world.

View Above Earth http://www.fourmilab.ch/earthview/vlatlon.html
At this site you can locate an aerial view of the earth at a specified latitude and longitude.

GPS Application 03[7]
Geocaching

Riverwalk Cache
by Cybercat & Eddie

N 29° 25.353 W 098° 29.314 (WGS84)
UTM: 14R E 549610 N 3254911
or convert to NAD27 at Jeeep.com

In Texas, United States [state map]
Hidden: 12/10/2002
Use waypoint: GCB39D (what's this?)

Note: To use the services of geocaching.com, you must agree to the terms and conditions in our disclaimer.
(ratings out of 5 stars. 1 is easiest, 5 is hardest)
Difficulty: ★★ Terrain: ★

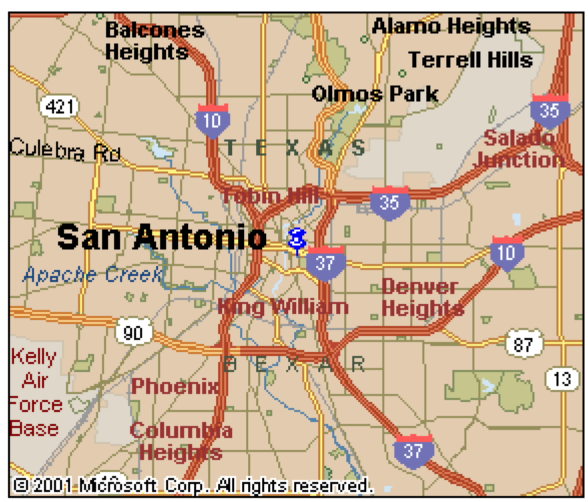
Virtual cache along famous Riverwalk

The traditional cache that was here has been plundered several times, so I am turning it into a virtual cache. I picked this location to get you right in the heart of the Riverwalk so you can see a lot of neat places that are here. The original cache was behind some bushes beside a wooden fence. To get credit for this cache, look behind the wooden fence and the stone wall and email me the answer to this question-What is on the other side of the stone wall? Do not put your answer on the Geocaching site.
The 2 difficulty rating is because you will probably have trouble keeping a clear signal on your GPS because of all the buildings. There is a big orange metal sculpture that you will see on your way to this cache - the Torch of Friendship - a gift from Mexico. The locals here either love it or hate it. Be sure and post what you think about it.

Additional Hints (No hints available**)**

Find...
...other caches hidden or found by this user
...nearby caches
...nearby placenames
...nearby benchmarks

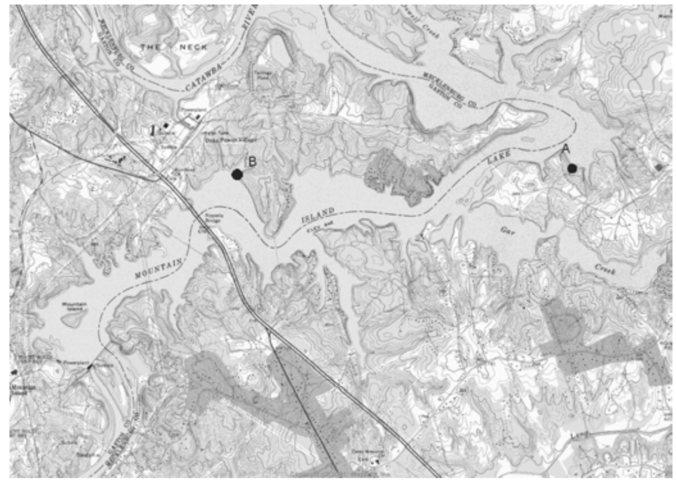
For online maps...
MapQuest Maps (BEST)
Topo Zone - topographical maps
(Site uses a different datum so coordinates may be off)
Microsoft Terra Server - satellite photos
Yahoo! Maps

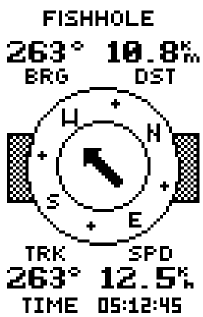


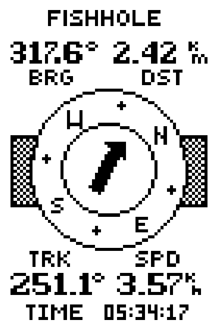
[7] URL: http://www.geocaching.com/seek/cache_details.aspx?ID=45981

GPS Application 04

A fisherman leaves the shore at point A to return to his favorite fishing hole at B on Mountain Island Lake. Equipped with a GPS unit, the fisherman sees the GPS navigation readout as shown on the next slide. Reading #1 was taken on shore at A and Reading #2 was taken on the lake enroute to point B. Interpret the readings and locate the fisherman's position at the second reading. What should the fisherman do to get back on course?







We know that AB is 10.8 km and that BP is 2.42 km. The difference in the bearing from A to B and the current track at P is 11.8°. Since PC is perpendicular to AB and AB is 97° West of North, we must have that PC is 7° West of North.

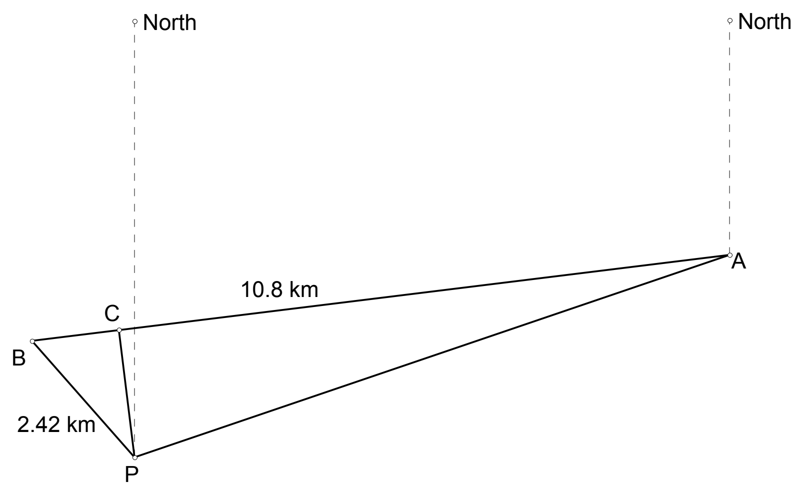
Thus, $\measuredangle BPC$ must measure 35.4°. Thus $\measuredangle B$ measures 54.6°.

So we need to use a little trigonometry to find several sides of these triangles.

$$PC = 2.42\cos(35.4°) = 1.97$$

$$BC = 2.42\sin(35.4°) = 1.40$$

This implies that AC = 9.4 and thus AP = 9.6 km. We also know that the fisherman should turn the boat by 317.6 – 251.2 = 66.4° to the right, until the track and the bearing are the same. He should continue on that track until the GPS unit shows 0 km.



Using GPS with Topographic Maps

James W. Kuhl

Objectives: Students will use a GPS receiver to determine the latitude and longitude of several objects and surface features. Students will plot these locations on a topographic map.

Tech Terms/Vocabulary: latitude, longitude, meridian, parallels, GPS, satellites, elevation, navigate, accuracy (terms specific to Garmin ***eTrex*** GPS receiver: normal skyview, advanced skyview, mark waypoint)

Materials: GPS receivers, topographic map of your location, Using GPS with Topographic Maps worksheet, pens OR pencils

Procedure:

1. Show how to turn on the GPS receiver.
2. Show how to scroll through the available pages.
3. Show where to find the data to be included in the chart **(latitude, longitude, elevation, accuracy, the person acting as the GPS Reader)** and explain the significance of the data. This part of the lesson should probably be done outside or in a location where the GPS can receive signals from orbiting satellites.
4. Give everybody a chance to practice: turning the GPS on and off; scroll through the pages; find and read the data needed for the chart. This is best accomplished by actually moving to various locations and having students compare the data they receive. Make sure to discuss the reasons for the differences in the data they observe (examples: **discrepant elevations – elevations are the most difficult values for GPS to calculate accurately. While standing at the edge of the Atlantic ocean a GPS calculated my elevation at 53 feet above sea level with an accuracy of 16 feet,** where they are standing in relation to one another, the differences in the accuracy being calculated and displayed on their GPS, etc.)
5. Once students are able to obtain the necessary data from the GPS allow them to visit the pre-selected locations and record the data on their worksheet.
6. After the data has been recorded have students plot the data points on their individual topographic maps.

Assessment: During the lesson as you arrive at a new location have different students read the required data from the GPS. Give each student a chance to read some portion of the required data o you are sure everyone knows how to locate the data using the GPS.

Evaluation: Data values on student worksheets should be consistent with those taken by the teacher and other students.

Using GPS with Topographic Maps

I. <u>Directions</u>: Read carefully to find the exact locations of the objects and surface features that are described in the chart. Use the GPS to find the required data. Record these data in the spaces provided in the chart. **<u>Make sure to include all appropriate units of measurement with the data.</u>**

Location	Latitude	Longitude	Elevation	Accuracy	GPS Reader
A. The corner of the bridge nearest to the school.					
B. The post of the football goal posts nearest the school.					
C. The regular door (not the garage doors) of the storage building by the middle school.					
D. The surveyor's stake at the bottom of the hill to the soccer field.					
E. The surveyor's stake at the top corner of the hill to the soccer field.					
F. The fire hydrant near the soccer field.					
G. The surveyor's stake at the top of the drainage ditch near the baseball backstop.					
H. The end pole of the backstop nearest the middle school.					

Location	Latitude	Longitude	Elevation	Accuracy	GPS Reader
I. The corner of the tennis court fence farthest from the middle school AND closest to Route 81.					
J. The maple tree between the baseball field and the tennis courts.					
K. The corner of the school on the same wall as our classroom windows (by the tar sidewalk).					

II. Directions: You will be using the data collected above to plot the objects and surface features on your topographic map. If there are other objects that you feel should be added to the topographic map use the GPS to record the data for the object in the chart below.

Location	Latitude	Longitude	Elevation	Accuracy	GPS Reader

Ask Dr. Math

Learning about GPS - posted May 8, 2000

A couple of years ago, I got my dad a GPS device. This is a handheld receiver that reads signals from satellites and can compute your exact location on the face of the earth. GPS stands for "Global Positioning System." It was developed by the United States military, but non-military people were allowed to have access to it. However, the non-military devices, like the one that I gave my dad, weren't as accurate as the ones that the military uses. Ours might have an error of up to 100 meters when computing position, while the military receivers would only show errors up to 22 meters. This was done by "scrambling" the signal that is read by the non-military devices.

One of the things I learned is that the satellites used in the system orbit at an altitude of 20,200 km above the surface of the earth. That's pretty high!

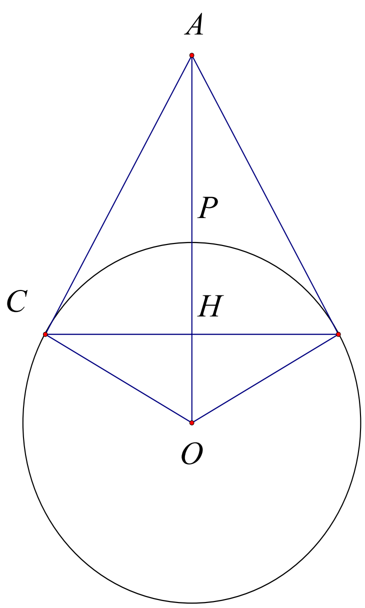
Here's the question I'd like you to answer. What percentage of the surface of the earth can each satellite see?

That sounds like a complicated question, but here is some useful information. Assume that the earth is a perfect sphere with a radius of 6374 km. You can find the formula for the area of a "spherical cap" in the Ask Dr. Math Sphere Formulas FAQ. Remember that the satellites are orbiting at 20,200 km.

Bonus: Are the GPS satellites "geosynchronous"? (Provide a definition of the word and explain how you know your answer is right.)

Solution:

The each satellite can see 38% of the earth's surface. First I drew tangents from the satellite to the earth. These tangents make a spherical cap on the earth's surface; this is the area that the satellite can see.



The earth is the circle and the satellite is the point A. The visual angle of the satellite is delimited by the two tangents to the circle from A. The tangent AC is at a right angle to the radius of the earth OC, therefore CAO is a right triangle. CH is the eight of CAO. The triangles CHO and CAH are similar, therefore:

AO:OC=OC:OH

AO is the sum of the earth's radius (OP) and the altitude of the satellite above the surface of the earth (AP), namely:

$$AO = OP + AP = 6374 + 20200 = 26574\ km$$

$$OH = \frac{OC * OC}{AO} = \frac{6374^2}{26574} = 1528.858$$

The height of the spherical cap (PH) is the difference between the earth's radius (OP) and OH, namely:

$$PH = OP - OH = 6374 - 1528.858 = 4845.142\ km$$

Calculate the surface "covered" by the satellite (Sc) using the formula:

$$Sc = 2\pi rh = 2\pi PO \times PH = 2\pi(6374)(4845.142) = 194,043,204.113\ km^2\ 2$$

The surface of the earth (Se) is given by the formula:

$$Se = 4\pi r^2 = 510,544,947.090\ km^2$$

The percentage of the surface of the earth can each satellite see is:

$$\frac{Sc}{Se} = \frac{194,043,204.113\ km^2}{510,544,947.090\ km^2} = 0.38$$

Bonus:

The geosynchronous orbit is the orbit synchronized with the sidereal rotation of the Earth (1 sidereal day is equal 23 h, 56, min, 4 s) with an inclination and eccentricity of zero. A satellite in this kind of orbit always appears in the same spot in the sky.

The orbital radius corresponding to this orbit can be found by equating centripetal acceleration to the acceleration due to gravity:

$$\frac{v^2}{r} = w^2 r = \frac{4\pi^2 r}{T^2} = \frac{MG}{r^2}$$

where v is the velocity, w is the angular velocity, T is the orbital period, M is the mass of the earth, and G is the universal gravitational constant. Solving for r gives:

$$r = \sqrt[3]{\frac{MGT^2}{4\pi^2}}$$

Plugging in $M = 5.976 \times 10^{24}\ kg$, $G = 6.672 \times 10^{-11}\ Nm^2 Kg^{-2}$, and $T = 1\ \text{day} = (23 \times 60 \times 60) + (56 \times 60) + 4 = 86164\ \text{sec}$ then gives:

$$r = 42168\ km.$$

Therefore, the distance from the earth's surface is 42168 - 6374 = 35794 km. The GPS satellites are orbiting at 20200 km, therefore they aren't geosynchronous.

According to Britannica Encyclopedia at 35,700 km, the period of orbit is 24 hours. Because this particular period is equal to the time the Earth rotates once, such a satellite travels at the same angular velocity as the surface of the Earth and appears to be stationary in the sky. This particular orbit is geosynchronous (or geostationary). Since the GPS satellites are 20,200 km above the surface of the earth, they are not geosynchronous.

Introduction to Latitude and Longitude

Lines of latitude circle the earth in an east/west direction while lines of longitude circumnavigate in a north/south direction from pole to pole. Degrees of latitude start at zero at the equator and increase to either ninety degrees north or ninety degrees south at the poles. An equal distance lies between each line of latitude (approximately 69 mile or 111 km), thus lines of latitude are often referred to as parallels. An equal degree of rotation lies between lines of longitude, which are sometimes referred to as meridians. However, since the circumferences of parallels vary with latitude (see the graticule below in Figure 1), the distance between lines of longitude varies from 69 miles at the equator to zero degrees at the poles. Hence, the number of miles per degree of longitude is a function of latitude.

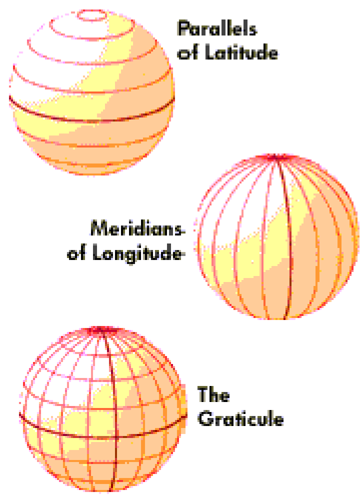


Figure 1: Latitude, longitude and graticule retrieved from:
http://www.geosys.com/cgi-bin/genobject/mapskills_latlong/tig5e6

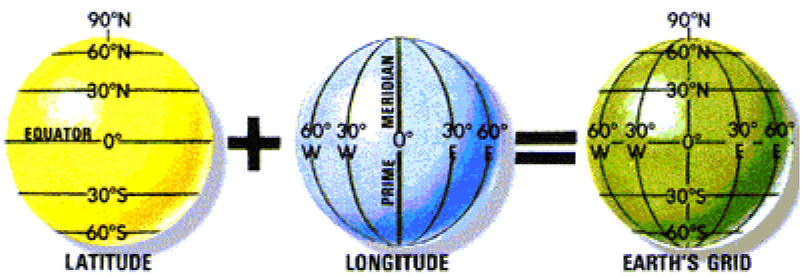


Figure 2: Latitude, longitude and earth's grid retrieved from:
http://www.hammondmap.com/latlong.html

Books

How GPS Works

Understanding the GPS: an Introduction to the Global Positioning System, by Gregory T. French

Understanding GPS: Principles and Applications, Elliott D. Kaplan

All About GPS: Sherlock Holmes' Guide to the Global Positioning System, Jerry Huang

Global Positioning System: Theory and Practice, B. Hofmann-Wellenhof, H. Lichtenegger & J. Collins

Understanding the Navstar: GPS, GIS, and IVHS, Tom Logsdon

Introduction to GPS: The Global Positioning System, Ahmed El-Rabbany

General GPS Receiver Use

GPS for Everyone: How the Global Positioning System Can Work for You, L. Casey Larijani

GPS Made Easy : Using Global Positioning Systems in the Outdoors, Lawrence Letham

A Comprehensive Guide to Land Navigation with GPS, Noel J. Hotchkiss

GPS/Topo Grid Guide, GPS Outfitters

GPS for Hiking

Using GPS: GPS Simplified for Outdoor Adventures, Bruce Grubbs

Basic Essentials Global Positioning Systems, by Scottie Barnes & Lafe Low

Wilderness Navigation: Finding Your Way Using Map, Compass, Altimeter, & GPS, by Bob Burns, Mike Burns & Paul Hughes

GPS Land Navigation: A Complete Guidebook for Backcountry Users of the NAVSTAR Satellite System, by Michael Ferguson, Randy Kalisek & Leah Tucker

GPS for Sailing and Boating

Using GPS, Conrad Dixon

Yachtsman's GPS Handbook: A Guide to the Global Positioning System of Satellite Navigation, Colin Jones

GPS Instant Navigation: From Basic Techniques to Electronic Charting, Kevin Monahan & Don Douglass

Coastal Loran and GPS Coordinates, Susie Stebbins & Rodney J Stebbins

GPS and Mathematics

The Mathematics of the Global Positioning System, Gail D. Nord, David Jabon, and John Nord, Mathematics Teacher, 90(6), Sept 1997, pp 455-464.

Mission Mathematics: 9-12, Peggy House, NCTM/NASA

Mathematics and the Global Positioning System, Floyd Vest, William Deidrich, and Kenneth Vos, HiMAP Pullout Section, Spring 1994, CoMAP.

Websites

Models & Resources for GPS Learning Activities – A. Ninno, OCM-BOCES

GPS: The New Navigation -- Nova Online at PBS

http://www.pbs.org/wgbh/nova/longitude/gps.html

"Ever wonder how the Global Positioning System (GPS) works? This extraordinarily complex system is actually based on some simple concepts. Find out what they are in this game. "

Satellites, Computers & Mapping at Ligon Middle School

http://www.ncsu.edu/midlink/gis/gis_intro.htm

"an interdisciplinary science & technology elective which encourages students to use GIS and GPS technology to solve problems."

Geocaching -- the sport where YOU are the search engine!

http://www.geocaching.com/

The Degree Confluence Project

http://www.confluence.org/

"The goal of the project is to visit each of the latitude and longitude integer degree intersections in the world, and to take pictures at each location. The pictures and stories will then be posted."

Buxley's Geocaching Waypoint -- clickable maps for geocachers!

http://www.brillig.com/geocaching/new_york.shtml

"Geocaching has quickly become an international sport. There are currently 8485 active caches around the world on every continent except Antarctica--and it's only a matter of time before someone places one there!"

Geographing -- take photos of locations with GPS coordinates

http://groups.yahoo.com/group/Geographing

"Geographing is the art of photographing a specific location then recording the precise GPS coordinates, time, date, bearing, tilt, weather, and other data. Geographing allows others to duplicate the photograph or panorama at a later date. Geographs are a way to document those special places on the planet for others to visit, enjoy, duplicate the photo with better equipment, compare historical changes, and share the experience of a special view.

GPS Waypoint Registry

http://www.waypoint.org/

"We are building a World Wide Database of GPS coordinates. Our Mission is to have a web site where people can exchange information about their travels. There is growing use of hand held and mobile GPS units. To help people use them and to share interesting places, we would like you to enter any waypoints you have collected. We also have helpful information about using your GPS and links to other sites you may find interesting."

Letterboxing North America -- like geocaching without a GPS receiver

http://www.letterboxing.org/index.html

"Letterboxing is an intriguing pastime combining navigational skills and rubber stamp artistry in a charming "treasure hunt" style outdoor quest. A wide variety of adventures can be found to suit all ages and experience levels.

Letterboxing at the Delano Middle School

http://www.delano.k12.mn.us/letterbox/letter.htm

Measuring the Earth -- An AskERIC Lesson Plan (stolen from Eratosthenes)

http://askeric.org/cgi-bin/printlessons.cgi/Virtual/Lessons/Mathematics/Geometry/GEO0004.html

"Eratosthenes, a Greek mathematician, was the first to measure the circumference of the earth.

GPS Walking -- Tourist Walks in the UK utilizing GPS map files for navigation

http://gpswalking.com/

"GPS Walking provides country walks in the Peak District. Each walk contains up to 150 color photographs, and selected video taken along the route. Unique to gpswalking is a Garmin MapSource and OziExplorer file which enables the walk to be uploaded to a GPS receiver which can then be used to guide the walker along the route with incredible accuracy. Each walk is carefully chosen for its charm and interest and all have been completed by families with young children (minimum aged seven). A comprehensive description of the route is included as well as all relevant hyperlinks and a brief history of the area. Post to: geographing@yahoogroups.com

NY Times Lesson Plan -- Where in the World? Understanding Latitude and Longitude in the Geography Classroom

http://www.nytimes.com/learning/teachers/lessons/19990512wednesday.html

"This lesson teaches students how to use longitude and latitude to locate various cities, regions, landforms, and bodies of water around the globe. Students use The New York Times Learning Network's crossword puzzle "Longitude and Latitude" to sharpen their atlas skills."

NY Times Lesson Plan -- Blazing Laptops: Using GPS Technology to Help Fight Fires
http://www.nytimes.com/learning/teachers/lessons/19981015thursday.html?searchpv=learning_lessons
"Students explore how digital maps and global satellite positioning are helping firefighters in the San Bernadino Valley. Students will then research what factors promote fire danger and various fire-fighting techniques."

Longitude & Latitude -- teaching resources
http://www.longitudelatitude.f2s.com/index.html
The Longitude and Latitude site is designed for three categories of people. It is primarily intended for students as a resource to supplement lessons taught in the classroom

Introduction to Latitude & Longitude -- National Geographic Lesson Plans
http://www.nationalgeographic.com/xpeditions/lessons/01/gk2/longlat.html
This lesson introduces students to latitude and longitude. They'll look at lines of latitude and longitude on a United States map and discuss the reasons why these lines are helpful.

The Mapping Theme Page -- Community Learning Network
http://www.cln.org/themes/mapping.html
"This "Theme Page" has links to two types of resources related to the study of Mapping. Students and teachers will find curricular resources (information, content...) to help them learn about this topic. In addition, there are also links to instructional materials & lesson plans.

USGS The Learning Web – lessons & teaching resources
http://www.usgs.gov/education/
http://www.usgs.gov/education/learnweb/wwmaps.html – Working with Maps

Find Your Longitude
http://www.pbs.org/wgbh/nova/longitude/find.html
"Want to understand why it is that having a precise timepiece (also known as a chronometer) helps you determine your longitude? Play this game, get lost on the high seas, and find out."

The Latitude & Longitude Story -- Putting it all Together & a Quiz!
http://academic.brooklyn.cuny.edu/geology/leveson/core/linksa/lat_long.html

Latitude Longitude Conversion – convert degrees/min/sec to decimal degrees, or reverse
http://www.directionsmag.com/latlong.asp

Geocache Models for Student GPS Activities

A Soldier's Pay
http://www.geocaching.com/seek/cache_details.asp?ID=13348

Back to School
http://www.geocaching.com/seek/cache_details.asp?ID=9596

Bear Down NGS marker
http://www.geocaching.com/seek/cache_details.asp?ID=12597

Compass Cache
http://www.geocaching.com/seek/cache_details.asp?ID=11641

Confluence Marks the Spot
http://www.geocaching.com/seek/cache_details.asp?ID=1698

Ding Dong School
http://www.geocaching.com/seek/cache_details.asp?ID=2676

GPS? IP Address!
http://www.geocaching.com/seek/cache_details.asp?ID=8978

History, Geography, Math and Recreation Project Geocache
http://www.geocaching.com/seek/cache_details.asp?ID=12706

History Cache - 1497
http://www.geocaching.com/seek/cache_details.asp?ID=9747

Hoisington NGS Benchmark
http://www.geocaching.com/seek/cache_details.asp?ID=12262

Latitude & Longitude
http://www.geocaching.com/seek/cache_details.asp?ID=6289

Map Datum Cache
http://www.geocaching.com/seek/cache_details.asp?ID=12911

Math Cache
http://www.geocaching.com/seek/cache_details.asp?ID=12773

McCoy School House
http://www.geocaching.com/seek/cache_details.asp?ID=6478

National Geodetic Survey Marker Cache
http://www.geocaching.com/seek/cache_details.asp?ID=12607

New Mexico NGS Benchmark Recovery
http://www.geocaching.com/seek/cache_details.asp?ID=12355

NGS Benchmark Recovery Cache
http://www.geocaching.com/seek/cache_details.asp?ID=11656

NGS Markers & GPS Accuracy Check
http://www.geocaching.com/seek/cache_details.asp?ID=10951

Oh Captain! My Captain!
http://www.geocaching.com/seek/cache_details.asp?ID=9056

Oklahoma NGS Benchmark Recovery Cache
http://www.geocaching.com/seek/cache_details.asp?ID=13372

Oregon History Lesson #02 - The Sunset Highway
http://www.geocaching.com/seek/cache_details.asp?ID=7909

Oregon History Lesson #03 - Largest Sitka Spruce
http://www.geocaching.com/seek/cache_details.asp?ID=7910

Oregon History Lesson #04 - aka I Hate I-5 XI
http://www.geocaching.com/seek/cache_details.asp?ID=10642

Oregon History Lesson #05 - Carto Cache
http://www.geocaching.com/seek/cache_details.asp?ID=11447

Oregon History Lesson #06 - McLoughlin
http://www.geocaching.com/seek/cache_details.asp?ID=9627

Oregon History lesson #07 - Wyam Falls
http://www.geocaching.com/seek/cache_details.asp?ID=7570

Oregon History Lesson #08 - Willamette Falls
http://www.geocaching.com/seek/cache_details.asp?ID=12713

Pennsylvania NGS Benchmark Recovery Cache
http://www.geocaching.com/seek/cache_details.asp?ID=12865

Round Lake National Natural Landmark
http://www.geocaching.com/seek/cache_details.asp?ID=1894

School House Rock – created by elementary students
http://www.geocaching.com/seek/cache_details.asp?ID=10549

Surveyor's Delight
http://www.geocaching.com/seek/cache_details.asp?ID=11141

Teacher's Cache
http://www.geocaching.com/seek/cache_details.asp?ID=1098

Teacher's Lesson
http://www.geocaching.com/seek/cache_details.asp?ID=7860

Teacher's Stash
http://www.geocaching.com/seek/cache_details.asp?ID=5322

The Klondike
http://www.geocaching.com/seek/cache_details.asp?ID=4545

Utah NGS Benchmark Recovery
http://www.geocaching.com/seek/cache_details.asp?ID=12351

Virtual Cache Mania I - Texas History
http://www.geocaching.com/seek/cache_details.asp?ID=5162

West Pike Fire Tower - USGS Marker Cache
http://www.geocaching.com/seek/cache_details.asp?ID=4588

Zebulon Ostrom's Legacy II
http://www.geocaching.com/seek/cache_details.asp?ID=7010