# HOMEWORK V
# MATH 527
# SPRING 2011

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**Problem 1.** (i) Let $p : E \longrightarrow X$ be the universal cover. Show that, for $n \geq 2$, the action of $\pi_1(X)$ on $\pi_n(X) \cong \pi_n(E)$ corresponds to the action of deck transformations on $E$.

(ii) Show that $\pi_1(\mathbb{RP}^n)$ acts trivially on $\pi_n(\mathbb{RP}^n)$ if and only if $n$ is odd.

**Problem 2.** Let $F \xrightarrow{i} E \xrightarrow{p} B$ be a fiber sequence of connected spaces.

(i) Construct an action of $\pi_1(E)$ on the higher homotopy groups of the fiber $F$. Hint: given $\alpha \in \pi_n(F)$ and $\gamma \in \pi_1(E)$, consider $p_*(\gamma \cdot i_*(\alpha)) \in \pi_n(B)$.

(ii) Find an example in which $E$ is simple, but $F$ is not. (You should be able to find an example in which $\pi_1(F)$ is not abelian.)

**Problem 3.** Let $X$ be a based space, let $n \geq 0$, and let $A$ be any abelian group. Show that there is a weak equivalence

$$\mathrm{Map}_*(X, K(A,n)) \simeq \prod_{0 \leq i \leq n} K\big(\tilde{\mathrm{H}}^i(X;A), n-i\big).$$

**Problem 4.** Determine the second Postnikov section $P_2(\Omega S^2)$. Show that each $P_n(\Omega S^2)$ is equivalent to a product $S^1 \times Y_n$, and describe the space $Y_n$. Show, however, that $\Omega S^2$ is not a Generalized Eilenberg-Mac Lane space (GEM).