

The following problems were selected from your textbook.

- Suppose there are 140 votes cast in an election among five candidates — Stein, O’Rourke, Cohen, Holt, and Massey — to be decided by plurality. After the first 100 votes are counted, the tallies are as follows:

| | |
|----------|----|
| Stein | 12 |
| O’Rourke | 23 |
| Cohen | 17 |
| Holt | 29 |
| Massey | 19 |

- What is the minimal number of remaining votes Holt needs to be assured of a win?
- What is the minimal number of remaining votes Cohen needs to be assured of a win?

- A campus programming committee must decide what kind of act to book for its next engagement. The choices are a comedian, a jazz trio, a pianist, a rock band, and a classical guitarist. The committee members decide to make the decision through an approval election and the resulting ballots are as follows:

| Number of Voters | 1 | 3 | 2 | 1 | 1 | 1 | 2 | 1 | 1 |
|-------------------------|---|---|---|---|---|---|---|---|---|
| Comedian | ✓ | | ✓ | | | | ✓ | ✓ | |
| Jazz trio | | | ✓ | ✓ | ✓ | | ✓ | | ✓ |
| Pianist | | | | ✓ | | ✓ | | | ✓ |
| Rock band | | | | | ✓ | | ✓ | ✓ | ✓ |
| Classical guitarist | | ✓ | | | | ✓ | | ✓ | ✓ |

Which act wins the vote?

3. The members of a community theater organization must vote to decide which play they would like to put on. The preference rankings of the members are as follows:

| Number of Voters | 2 | 1 | 4 | 1 | 1 | 3 | 1 | 2 |
|---------------------|---|---|---|---|---|---|---|---|
| The Fantasticks | 1 | 1 | 2 | 3 | 4 | 2 | 3 | 4 |
| Romeo and Juliet | 3 | 4 | 1 | 1 | 1 | 4 | 2 | 3 |
| Our Town | 4 | 3 | 3 | 4 | 2 | 1 | 1 | 2 |
| Death of a Salesman | 2 | 2 | 4 | 2 | 3 | 3 | 4 | 1 |

- (a) Which play would win a plurality vote?
 - (b) Which play would win a plurality vote with a runoff between the top two finishers?
 - (c) Which play would win under Borda's Method?
 - (d) Which play, if any, is the Condorcet winner?
 - (e) Could the three voters who ranked Our Town first and The Fantasticks second achieve a preferable outcome in an election decided by Borda's method by voting strategically if the others voted as shown in the table?
 - (f) Could the three voters who ranked Our Town first and The Fantasticks second achieve a preferable outcome in an election decided by the plurality method with a runoff between the top two by voting strategically if the others voted as shown in the table?
4. Explain why the plurality method with a runoff between the top two finishers satisfies the Pareto optimality property.
5. Construct an example of preference rankings for an election with four candidates — A, B, C, and D — so that, in two-person races, A would defeat B, B would defeat C, C would defeat D, and D would defeat A.

6. Partial results of Greenland's Inatsi-satut election of March 5, 1995 are shown in the table. Apportion 28 seats to the three political parties based on these results, using
- (a) Hamilton's method
 - (b) Lowndes' method
 - (c) Jefferson's method
 - (d) Webster's method

| Party | Siumut | Inuit Ataatigiit | Atassut |
|----------------------------|--------|------------------|---------|
| Percentage of Votes | 38.5 | 20.3 | 29.7 |

7. The 1997 enrollments of the junior high schools in the Chino Valley Unified School District in California are shown in the table. Suppose the district receives a grant to purchase 225 new computers for its five junior high schools. Apportion the computers to the schools based on their enrollments, using
- (a) Hamilton's method
 - (b) Lowndes' method
 - (c) Jefferson's method
 - (d) Webster's method
 - (e) Which apportionment is least favorable to the largest school, Canyon Hills?
 - (f) Do any of these apportionments violate the Quota Property?

| School | Canyon Hills | Magnolia | Ramona | Townsend | Woodcrest |
|-------------------|--------------|----------|--------|----------|-----------|
| Enrollment | 1050 | 924 | 917 | 841 | 502 |

8. If the natural divisor works for Webster's method, explain why Webster's method and Hamilton's method give the same apportionment.

9. Explain why Jefferson's method will never give a state fewer seats than its natural quota rounded down.