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1. The student council is electing a new president and vice-president. The preference rankings of each voter and the candidates of whom they approve are shown below.

| Number of Voters | $\mathbf{1 1}$ |  | $\mathbf{9}$ | $\mathbf{3}$ | $\mathbf{7}$ |  |
| :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| Applegate | 1 | $\checkmark$ | 3 |  | 2 |  |
| Bryant | 2 | $\checkmark$ | 2 | $\checkmark$ | 1 | $\checkmark$ |
|  | 1 | $\checkmark$ |  |  |  |  |
| Chancellor | 3 |  | 1 | $\checkmark$ | 3 |  |

Suppose the first-place finisher is the new president and the second-place finisher is the new vice-president. Who are the new president and vice-president if the winners are selected by:

- the plurality method?
- Borda's method?
- the approval voting method?

On June 22, 2000, Frank Morgan asked readers of his "Math Chat" column to design their own voting system (http://www.maa.org/features/mathchat/mathchat_6_22_00.html). In response, Joseph DeVincentis proposed a modification of the approval voting system in which a voter has three options for each candidate: approval, no opinion, or disapproval. A candidate earns 1 point for each "approval" vote, 0 points for each "no opinion" vote, and -1 points for each "disapproval" vote. The candidate earning the most points wins.
The table below contains approval (A), disapproval (D), and no opinion (NO) votes. Who is the new president according to DeVincentis' system of tabulating votes?

| Number of Voters | $\mathbf{1 1}$ | $\mathbf{9}$ | $\mathbf{3}$ | $\mathbf{7}$ |
| :---: | :---: | :---: | :---: | :---: |
| Applegate | A | D | NO | D |
| Bryant | A | A | A | A |
| Chancellor | D | A | NO | NO |

2. Which of universal domain, Pareto optimality, non-dictatorship, and independence from irrelevant alternatives does DeVincentis' system satisfy? Which of these properties does DeVincentis' system not satisfy? Justify your answers.
