History of Mathematics #4

1. Before 11 pm, Sunday, February 5. Go to the Forum "Archimedes" and make at least one substantive contribution by 11 pm, Sunday, February 5, and at least one substantive response to others' postings before class on Tuesday, February 7. Write about the following:

   (a) Where in the K–16 curriculum do the various results of Archimedes discussed in Dunham make their appearance?

   (b) Where in the K–16 curriculum do the proofs of these results make their appearance?

   (c) Do you think we should present Archimedes' proof of the area of a circle in middle school? in high school?

2. Before Tuesday, February 7.

   (a) Read Dunham Chapter 4, MTA Sketch 7, and Boyer Chapter 8. If you aren't able to to read all of Boyer, at least read the sections in Chapter 8 on *The Method*, Volume of a sphere, and Recovery of *The Method*.

   (b) Think about the following questions for discussion at the Centra session:

      i. Study Euclid's proof by exhaustion of the volume of a circular cone (XII.10). Refer also to XII.7 and XII.5. Pretty amazing!

      ii. On pages 93–94 of Dunham it is asserted that the perimeter of a regular polygon inscribed in a circle is less than the circumference of the circle, and also that the perimeter of a regular polygon circumscribed about a circle exceeds the circumference of the circle. How might these statements be justified?

      iii. Try making and measuring some perimeters of inscribed and circumscribed polygons using Wingeom or Geometer's Sketchpad.

      iv. Explain why the derivative of the area formula for a circle equals the circumference formula, and the derivative of the volume formula for a sphere equals the surface area formula.

      v. Can you find any of Archimedes' works online?

      vi. How did you first see the formula for the volume and the surface area of a sphere formally proved?

      vii. Look at the website www.geocities.com/CapeCanaveral/Lab/3550/pimnem.htm to see some mnemonics for memorizing π.

viii. Study Archimedes method for discovering the volume of a sphere as described in Boyer. How does this relate to the derivation of the volume of a sphere that we did in our summer geometry class using Cavalieri's principle?

ix. Find a more detailed description of *The Method* to discover the formula for the area of a sector of a parabola—an amazing non-physical example of Archimedes using the power of a lever!

3. Tuesday, February 7, 7–9 pm. Attend the Centra session to discuss the readings, forum, and comments and questions on the assigned homework due on Friday.

4. Homework problems due Friday, February 10, 11 pm, uploaded on the moodle site (preferred method) or submitted to the email address mathhist@ms.uky.edu.

   (a) Take a circular cone with base radius r and height h. Regard it as hollow and remove the base (like those conical paper cups that are often found in dispensers). The lateral surface area can be determined by making a cut along the slant height of the cone and flattening it out into a circle with a wedge removed. Use this idea to derive the formula for the lateral surface area in terms of r and h. Do not use outside sources for this problem, though you may talk with each other.

   (b) Derive the formula for the volume of a sphere by slicing using calculus. It is permitted to use outside sources for this problem.

   (c) Look up and present a calculus derivation for the surface area of a sphere. It is permitted to use outside sources for this problem.

   (d) Use trigonometry and a calculator to calculate the length of a chord subtending an angle of 1 degree in a circle of radius $60p$ and verify Dunham's statement at the bottom of page 106 that it is approximately $1.0472p$. Do not use outside sources for this problem, though you may talk with each other.

   (e) i. Use the Bailey-Borwein-Plouffe formula for p,

$$\pi = \sum_{n=0}^{\infty} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) \left(\frac{1}{16} \right)^n$$

   to approximate the first 9 decimal digits of π. How many terms do you need? You might want to use a spreadsheet.

   ii. Why is this formula (also known as the BBP formula) known as a "digit extraction" formula? See, for example, mathworld.wolfram.com.

(f) Extra Credit. We now know that if we begin with a sphere S of radius r and circumscribe around it a cylinder C of radius r and height $2r$, then the surface area of S equals the lateral surface area of C. What is perhaps more surprising is the fact that if we project any region of the surface of the sphere onto the cylinder, using directions perpendicular to the cylinder, that the projected region will have the same area as the original region. Can you provide an explanation of this? You will likely need to use calculus. Are there any maps of the earth that are made using this area-preserving projection? It is permitted to use outside sources for this problem.