History of Mathematics #7

1. Before 11 pm, Sunday, February 26. Go to the Forum "Calculus" and make at least one substantive contribution by 11 pm, Sunday, February 26, and at least one substantive response to others' postings before class on Tuesday, February 28. Write about the following:

   How is or can the groundwork be laid in the K–12 curriculum for a deep conceptual understanding of the notion of derivative? What can we learn from the historical development in this respect? Think about, for example, lines, slope, distance and velocity, functions, algebra, formulas with several variables in which one or more are held constant as others vary, and limits. What am I forgetting/neglecting?

2. Before Tuesday, February 21.

   (a) Read Dunham Chapter 7 and MTA Sketches 13, 16 and 18. Skim Boyer Chapters 16–19.

   (b) Think about the following questions for discussion at the Centra session:

      i. Carefully study the binomial theorem and its deployment in the estimation of π.

      ii. What is the relationship of the binomial theorem to Taylor series?

      iii. Plot the function $\sqrt{1-x}$ and the first 6 partial sums for this function given on page 170 of Dunham. Repeat for the function $(1+x)^{-3}$.

      iv. The chapter in Dunham is a bit light on some of the mathematical developments setting the stage for calculus. In fact, Dunham has written a fine, but technical book, devoted to the history of calculus. Explore some other sources to try to better understand the invention and development of calculus. In particular, learn more about the role of the Fundamental Theorem of Calculus. Are there any particularly good websites to recommend?

      v. How can you motivate the Fundamental Theorem of calculus graphically? How about some of the theorems, such as the sum and product rules for differentiation?

      vi. Practice deriving some of the basic formulas for differentiation and integration.

3. Tuesday, February 28, 7–9 pm. Attend the Centra session to discuss the readings, forum, and comments and questions on the assigned homework due on Friday.

4. Homework problems due Friday, March 3, 11 pm, uploaded on the moodle site (preferred method) or submitted to the email address mathhist@ms.uky.edu.

   (a) (Ptolemy knew about this one!) Use a program like Wingeom or Geometer's Sketchpad to draw a circle of diameter 1. Inscribe any triangle ΔABC in the circle. Measure angle A and the length of the chord $\overline{BC}$. Make a conjecture on the precise relationship between these two measures; i.e., come up with a formula for BC in terms of A. (Move the points around the circle, including some "nice" angles A.) Are you surprised? Prove your conjecture.

   Do not use outside sources for this problem, though you may talk with each other.

   (b) Descartes was interested in deriving algebraic expressions for curves defined by certain properties. Consider the curve that is defined as the locus of points (x, y) for which the sum of the distances to the points $(-3, 0)$ and $(3, 0)$ equals 10. Start with this definition and derive an equation for this curve in the form $ax^2 + bxy + cy^2 + dx + ey + f = 0$. What kind of curve is this?

   Do not use outside sources for this problem, though you may talk with each other.

   (c) Let's try to understand Pascal's triangle a bit. First *define* $c_{n,r}$ to be the coefficient of x^r in the expansion of $(1+x)^n$, $n = 0, 1, 2, \ldots$, $r = 0, 1, 2, \ldots, n$. I know that you know the formulas for $c_{n,r}$, but for the moment FORGET THAT YOU KNOW THEM!

      i. Based only on the definition above, explain why $c_{n,0} = 1$, $c_{n,n} = 1$, and $c_{n,1} = n$.
      ii. By thinking about the coefficient of x^r in $(1 + x)^{n-1}(1 + x)$, explain why $c_{n,r} = c_{n-1,r-1} + c_{n-1,r}$.
      iii. By thinking about the coefficient of x^r in $(1 + x)(1 + x) \cdots (1 + x)$ (n terms) explain why $c_{n,r}$ must equal the number of ways of choosing a subset of size r from a subset of size n, even if you don't know the formula for either quantity.

   Do not use outside sources for this problem, though you may talk with each other.

   (d) Euclid and Fermat proved some theorems about perfect numbers (see "Number(s), perfect" in the index to Dunham). Prove that if $2^n - 1$ is a prime, then $2^{n-1}(2^n - 1)$ is perfect. Try not use outside sources for this problem, but if you run into obstacles, then you may.

   (e) The Taylor series expansion of a function $f(x)$ about the point $x = 0$ is given by

$$f(x) = \frac{f(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} + \frac{f^{(3)}(0)x^3}{3!} + \cdots + \frac{f^{(k)}(0)x^k}{k!} + \cdots$$

Use this formula to obtain the binomial theorem expansion of $(1+x)^{\frac{m}{n}}$ given on page 168 (where Q is used instead of x). Try not use outside sources for this problem, but if you run into obstacles, then you may.