History of Mathematics #9

1. Before 7 pm, Tuesday, March 21. Go to the Forum "Curves" and make at least one substantive contribution by 7 pm, Tuesday, March 21. Write about the following:

   Many curves have interesting names and interesting historical backgrounds. For example, we have already heard about the Witch of Agnesi, the brachistochrone, the cycloid, etc. Find one specific example of a curve (e.g., cycloid), and (1) give a brief description of how it is generated or described (e.g., curve traced by a point on the perimeter of a circle rolling along a straight line), and (2) some history about it (e.g., who studied it and when, and what mathematics it relates to). DO NOT DUPLICATE A CURVE THAT SOMEONE ELSE HAS ALREADY POSTED OR THAT WE HAVE ALREADY ENCOUNTERED IN THE COURSE! If you can create constructions with Geometer's Sketchpad, Wingeom, or other graphing software, I would like to share some of these during the Centra session.

2. Before Tuesday, March 21.

   (a) Read Dunham Chapter 8 and MTA Sketches 21 and 22. Skim Boyer Chapter 20.

   (b) Think about the following questions for discussion at the Centra session:

      i. What did Huygens' clock have to do with the cycloid?

      ii. What is the "Calculus of Variations" and what does this have to do with the brachistochrone?

      iii. How is the formula for the sum of the reciprocals of the positive integers derived?

      iv. What precisely does it mean for $1/3$ to equal $0.333\cdots$?

      v. How is the sum of the geometric series derived?

      vi. What are various proofs of the divergence of the harmonic series described in Dunham? What aspects of these proofs are the same, and what are different?

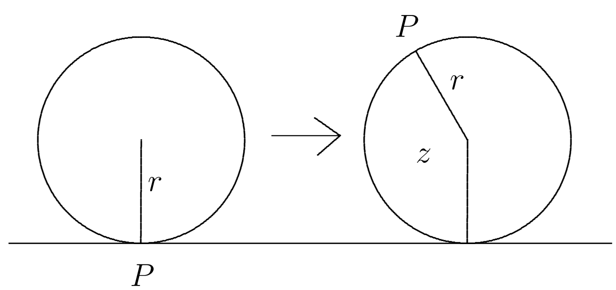
      vii. What is the Law of Large Numbers mentioned in MTA Sketch 21?

      viii. When was life insurance invented?

3. Tuesday, March 21, 7–9 pm. Attend the Centra session to discuss the readings, forum, and comments and questions on the assigned homework due on Saturday.

4. Homework problems due Saturday, March 25, 11 pm, uploaded on the moodle site (preferred method) or submitted to the email address mathhist@ms.uky.edu.

(a) Do not use outside sources for this problem, though you may talk with each other. Derive the formula of a cycloid in the following way: Assume that a circle of radius a is rolling on the x-axis—see the diagram on page 185 of Dunham. Initially the circle is centered at $(0, a)$ and there is a point P marked on the circle that is initially at $(0, 0)$. Let r be the line segment from the center of the circle to the point P. As the circle rolls on the x-axis, let z be the angle, in radians, that P has rotated clockwise about the center of the circle so far—the angle that r makes with respect to a vertical line through the center of the circle.



Prove that $P = (x, y)$ traces out a curve with equations

$$\begin{aligned} x &= az - a\sin z \\ y &= a - a\cos z \end{aligned}$$

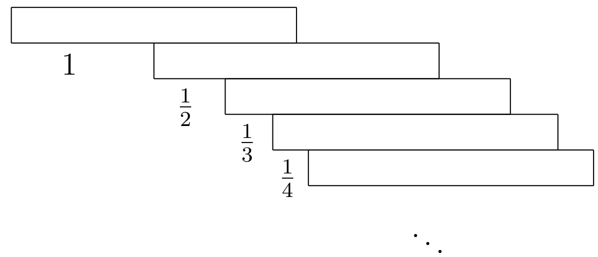
(b) You may consult outside sources for this problem.
Briefly describe the "tautochrone." What property does this curve have, who discovered this property, and what does this have to do with the chapter in Dunham?

(c) Do not use outside sources for this problem, though you may talk with each other.
Here is an interesting application of the harmonic series.

i. Prove by induction that for all positive integers k,

$$1 + (1 + 1) + (1 + 1 + \tfrac{1}{2}) + (1 + 1 + \tfrac{1}{2} + \tfrac{1}{3}) + \cdots + (1 + 1 + \tfrac{1}{2} + \tfrac{1}{3} + \cdots + \tfrac{1}{k}) \\ = (k + 1)(1 + \tfrac{1}{2} + \tfrac{1}{3} + \cdots + \tfrac{1}{k+1}).$$

ii. Consider the following overhanging stack of $n + 1$ bricks, each of length 2, where the numbers indicate the amount of overhang:



Let's take the following as facts:

A. Each brick has length 2.

B. The (x-coordinate of the) center of mass of each individual brick is at its midpoint.

C. For every $k \leq n+1$ the net center of mass of the top k bricks is the average of the locations of their individual centers of mass.

D. If for every $k < n+1$ the net center of mass of the top k bricks does not lie to the left of the left end of brick $k+1$, then the bricks will not topple.

Prove that the bricks will not topple if the left ends of the bricks (starting from the top) are at positions 0, 1, $1+\frac{1}{2}$, $1+\frac{1}{2}+\frac{1}{3}, \ldots, 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$, as indicated in the diagram.

iii. Conclude that it is possible to create a stable tower of bricks overhanging as far to the left of the base as you wish!

(d) Do not use outside sources for this problem, though you may talk with each other. Here's a little fun in probability theory. Assume that you have three cubical dice, A, B, and C. (I guess in school they now call these things "number cubes.") The sides of die A are labeled with the numbers 1, 2, 11, 12, 15, and 16. The sides of die B are labeled 5, 6, 9, 10, 13, 14. The sides of die C are labeled 3, 4, 7, 8, 17, 18. Two players each select one of the dice and roll once. For each pair of dice, calculate the probability that one die beats the other. Are you surprised?