The Place of Mathematics and the Mathematics of Place

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My Personal Place

By way of introduction:

What contributed to my inclination toward teaching and mathematics?

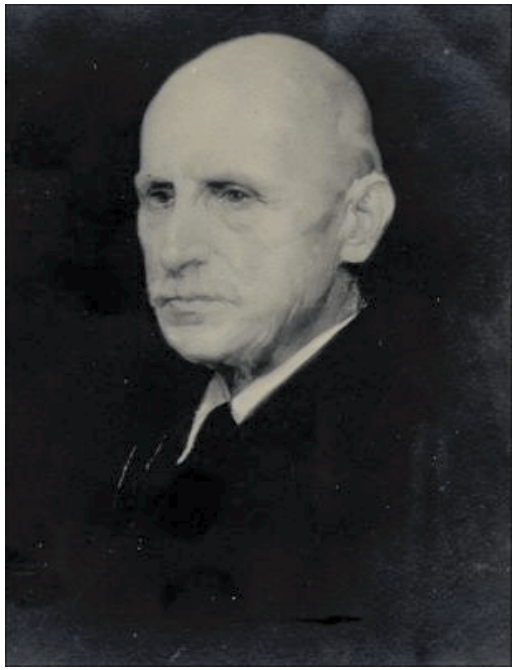
My Personal Place

A family (over)loaded with academics: father, mother, aunts, uncle, grandfather, great aunt, great-grandfather, great-aunt, great-great-grandfather, etc.

It was therefore easy to envision what it meant to teach at a high school or university level.

My Personal Place

Ernest G. Sihler—Brother of my great-grandmother



Classics scholar in the group of the first four Ph.D.'s awarded by Johns Hopkins (1878). Asked by Sylvester to provide a Greek motto for the new *American Journal of Mathematics* (Proof of things not seen—was this ever used?)

My Personal Place

Gustav Strube—My great-grandfather

First conductor of the Baltimore Symphony Orchestra (1916).
Faculty member at the Peabody Institute.

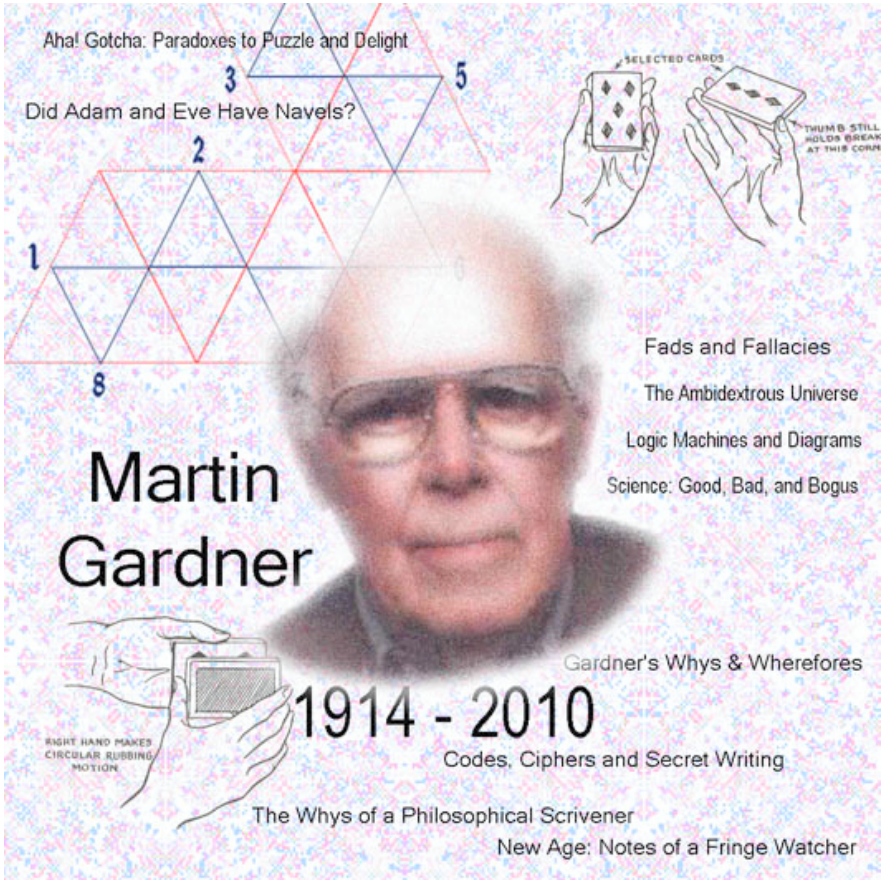
My Personal Place

Andrew H. MacPhail—My grandfather

Ph.D. Brown (1923). Professor of Educational Psychology at Brown. Studied intelligence and psychology tests for selection and evaluation of college students. Served on the College Entrance Commission, which developed the Scholastic Achievement Test.

My Personal Place

My father's (Frederick Lee) home collection of recreational (and nonrecreational) math books, including those by Martin Gardner



My Personal Place

A spectacular set of K–12 teachers in the Baltimore County school system

My Personal Place

Being raised in a faith community (Bahá'í) that explicitly acknowledges the presence of tremendous human capacity and the high station of the teacher who nurtures it

Dost thou reckon thyself only a puny form
When within thee the universe is folded?

My Personal Place

This is not meant as justification of personal worth due to pedigree; rather

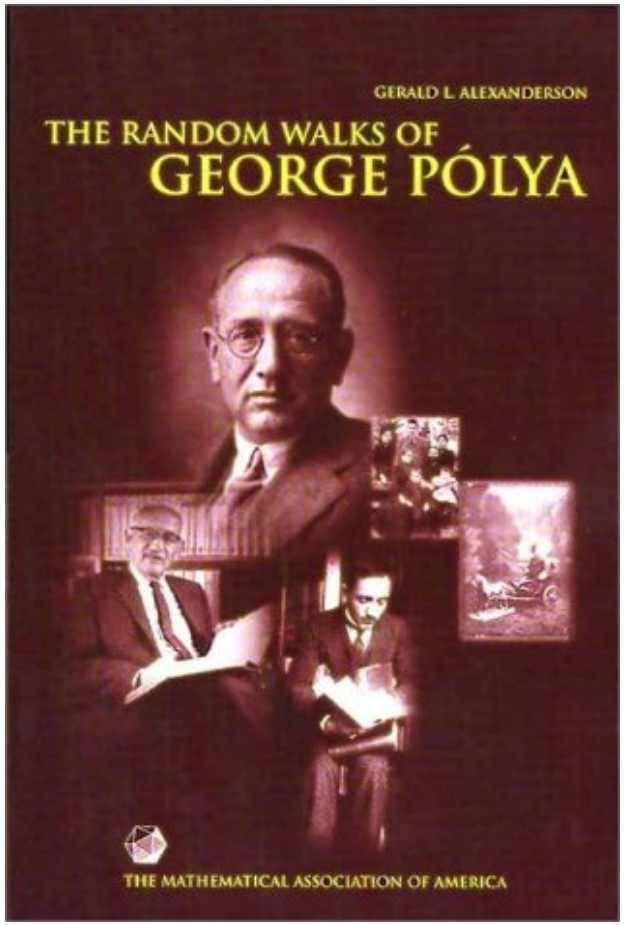
Recognize the potentially significant role of personal place, including context and informal education, in the development of students' capacity

The Place of the Classroom

As a K–12 student, I often engaged in math classes at a high cognitive level as a result of a teacher's direct instruction ("lectures"). As a teacher I quickly learned that most of my students were not similarly engaged by this process. Not all developed their "mathematical habits of mind" or "mathematical practices" through my in-class lectures and out-of-class homework (often worked on individually).

The Place of the Classroom

George Polyá — Let Us Teach Guessing



Raising the explicit awareness of and cultivating mathematical practices among your students

The Place of the Classroom

Promote and observe struggle

Deliberately create opportunities in the classroom in which students grapple with mathematics and communicate with each other. Carefully listen and use what you learn to shape what is to come. Provide an environment in which mistakes are opportunities for learning and not censure.

The Place of the Classroom

There are, it seems, two muses: the Muse of Inspiration, who gives us inarticulate visions and desires, and the Muse of Realization, who returns again and again to say "It is yet more difficult than you thought." This is the muse of form. It may be then that form serves us best when it works as an obstruction, to baffle us and deflect our intended course. It may be that when we no longer know what to do, we have come to our real work and when we no longer know which way to go, we have begun our real journey. The mind that is not baffled is not employed. The impeded stream is the one that sings.

—Wendell Berry

The Place of the Classroom

One measure of the efficacy of instruction (or professional development) is to determine what persists (mathematical content and mathematical practices) within a student or participant one year later.

The Place of Cognitive Development

Lev Vygotsky's Zone of Proximal Development (ZPD)

"[t]he distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers."

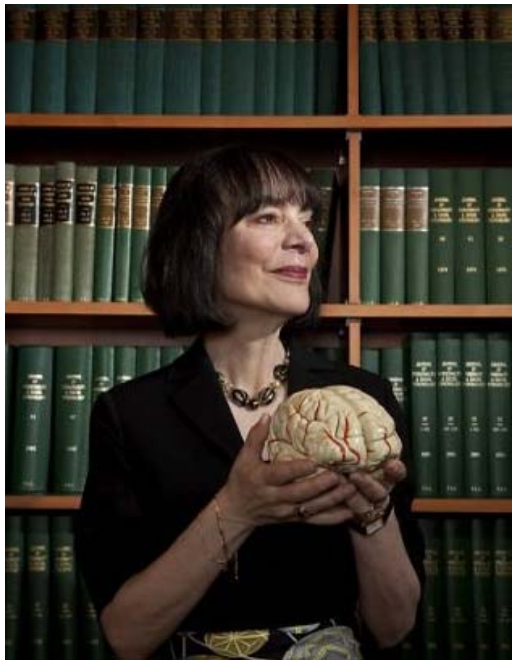
The Place of Cognitive Development

Vygotsky's Zone of Proximal Development (ZPD)

Learning can be promoted when the material is above the student's current state, but not so far above to be unattainable even with scaffolding and assistance

The Place of Cognitive Development

The Growth Mindset



See the research of Carol Dweck, and article
`http://www.ms.uky.edu/~braun/Braun_Beliefs_2012.pdf`.

The Place of Cognitive Development

London Taxi Drivers' Brains

`http://video.nationalgeographic.com/video/london-taxi-sci`

Brains can continue to grow significantly.

The Place of Cognitive Development

"praise focused on developing malleable intelligence beliefs positively affects subsequent student achievement, while praise that cultivates fixed intelligence beliefs has the opposite effect."

With the explicit knowledge and understanding of your students, foster a growth mindset rather than a fixed mindset in your class

The Place of Mathematics

On the one hand

Many value mathematics precisely because it transcends place, even though it may be initially motivated by a particular context.

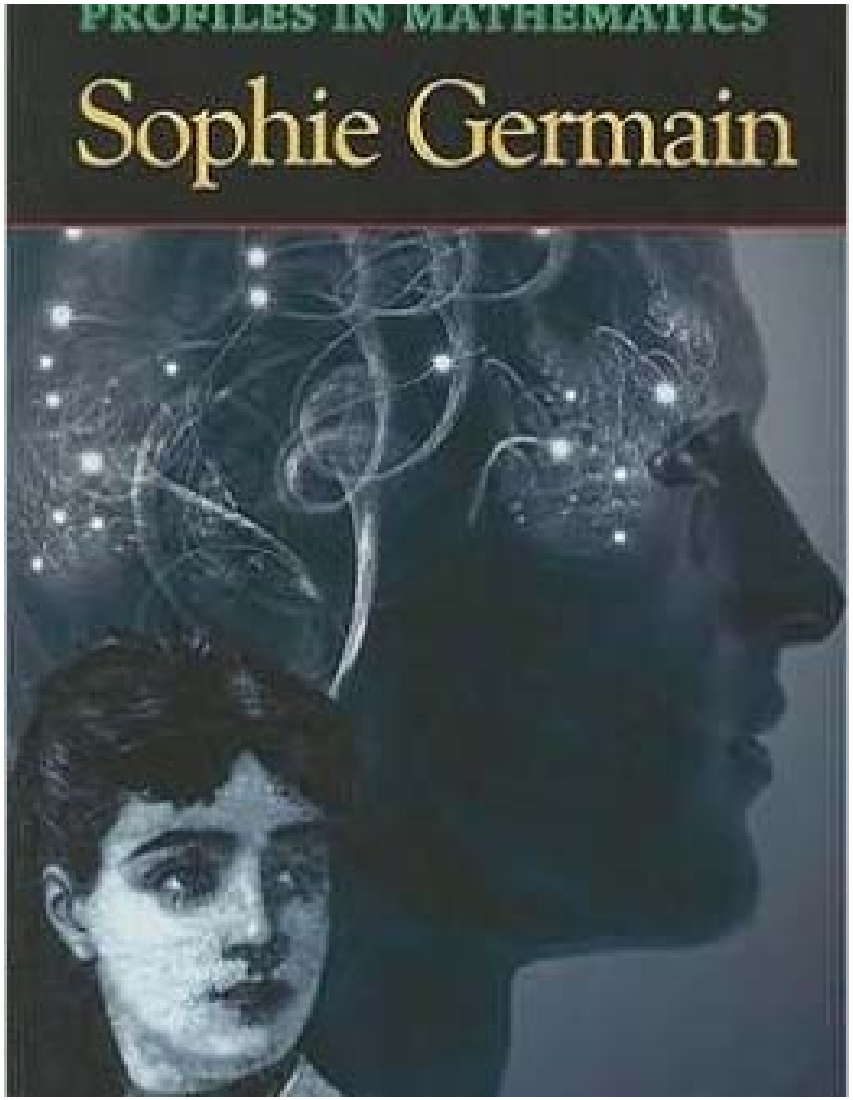
"Without doubt, all mathematical development has its psychological roots in more or less practical requirements. But once started under the pressure of necessary applications, it inevitably gains momentum in itself and transcends the confines of immediate utility." (Courant and Robbins, *What is Mathematics?*)

"The essence of mathematics lies in its freedom." (Georg Cantor, in *On Mathematics,* by Moritz)

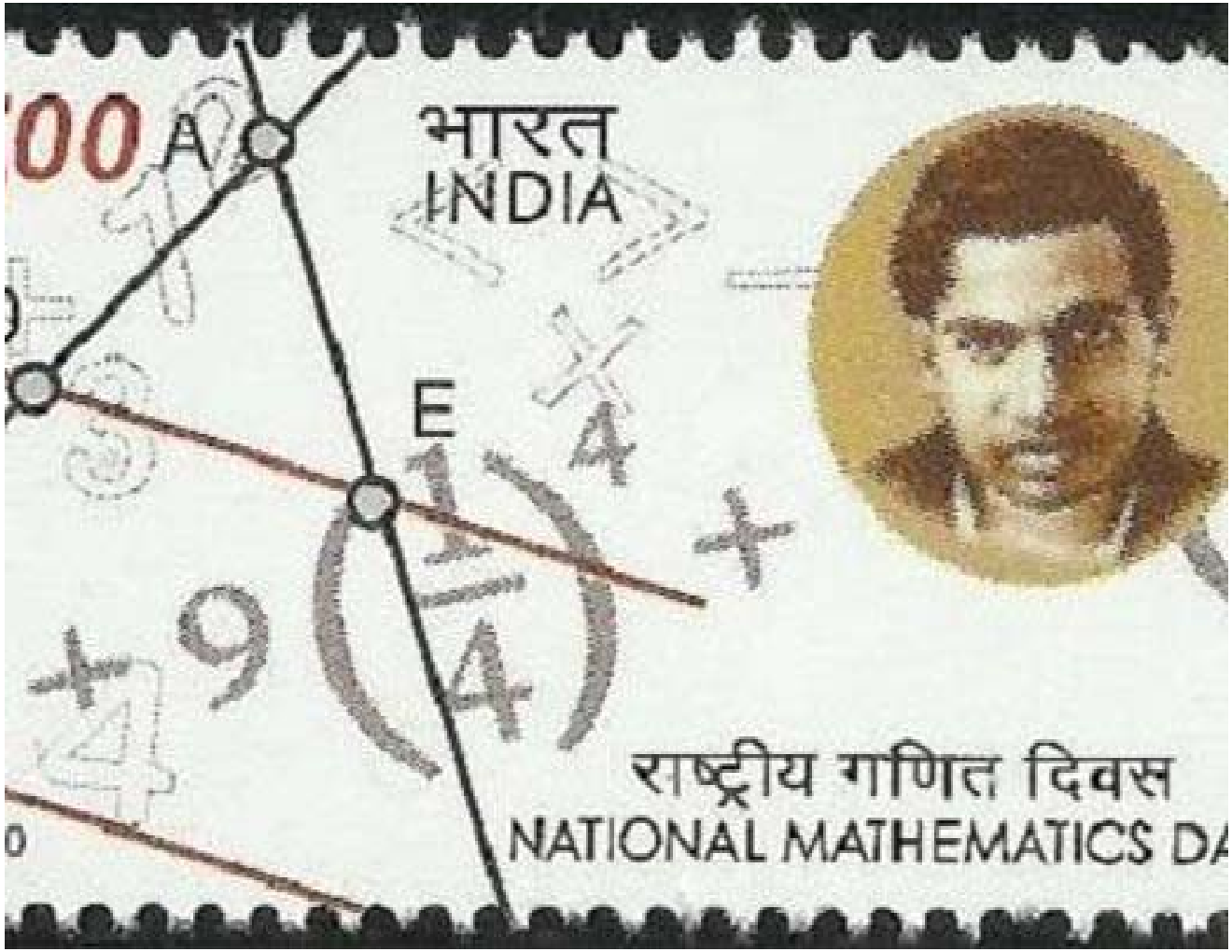
The Place of Mathematics

It is possible to deeply engage in significant mathematics with little more than paper and pencil.

The Place of Mathematics



The Place of Mathematics



Srinivasa Ramanujan

The Place of Mathematics

Abstract (from dictionary.com)

Adjective:

- Thought of apart from concrete realities, specific objects, or actual instances: "an abstract idea."
- Expressing a quality or characteristic apart from any specific object or instance, as "justice, poverty, and speed."
- Theoretical; not applied or practical: "abstract science."
- Difficult to understand; abstruse: "abstract speculations."

Verb: To consider as a general quality or characteristic apart from specific objects or instances: "to abstract the notions of time, space, and matter."

The Mathematics of Place

On the other hand

The value of place (including personal, social, geographic, and community place) offers a rich and meaningful setting in which to nurture the understanding of mathematics.

"Mathematics stands forth as that which unites, mediates between Man and Nature, inner and outer world, thought and perception, as no other subject does." (Friedrich Froebel, in *On Mathematics,* by Moritz)

The Place of Mathematical Work

There is a continuum of participants and stakeholders in STEM education: P-12 students, school teachers, counselors, principals, superintendents, parents, community members, college students taking math and science courses, majors in STEM fields, aspiring STEM teachers, higher education faculty in content departments teaching all of these types of students, higher education faculty in education departments teaching courses for future teachers and engaged in teacher training programs, practicing teachers including those who supervise student teachers or are enrolled in graduate programs, higher education faculty engaged in STEM education research or in outreach to schools, and various local, state, regional, and national agencies and organizations, public and private, commercial and non-profit. There is a natural tendency for the diverse participants to operate each within a somewhat limited sphere of activity.

The Place of Mathematical Work

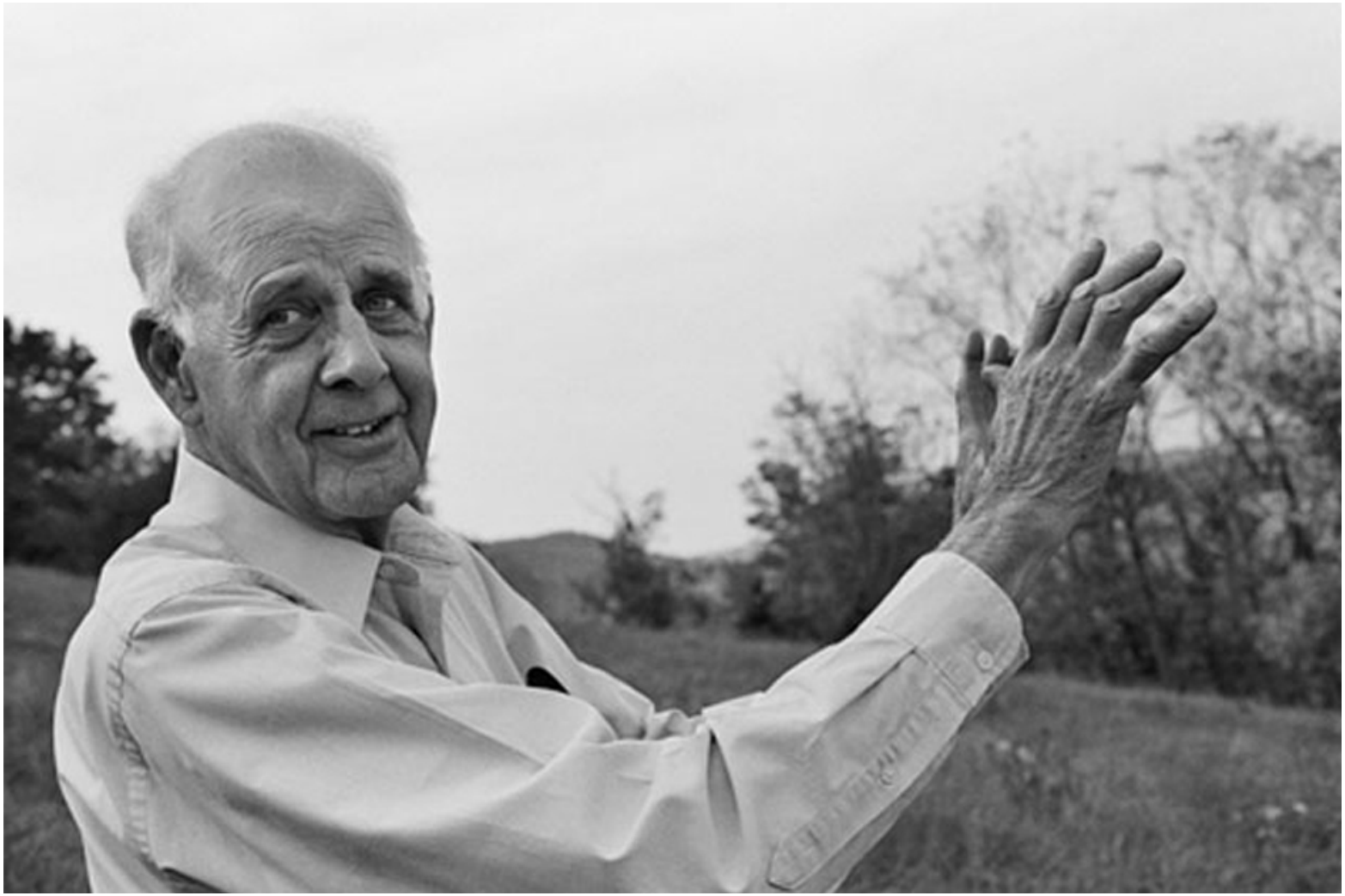
There is an imperative need for mathematicians to lend their expertise to this continuum, and for institutions to appropriately reward their contributions

One example of guidance: *The Mathematics Education of Teachers II*, CBMS

The Place of Central Appalachia

What do you think of when you think of Appalachia?

The Place of Central Appalachia



The Place of Central Appalachia

What We Need Is Here

Geese appear high over us,
pass, and the sky closes. Abandon,
as in love or sleep, holds
them to their way, clear
in the ancient faith: what we need
is here. And we pray, not
for new earth or heaven, but to be
quiet in heart, and in eye,
clear. What we need is here.

—Wendell Berry

The Place of Central Appalachia

It is important to note that reading the social reality of a population from within is different than studying it as an outsider. In instances where the population in question is relatively poor in material resources, outsiders with access to greater means frequently see only deprivation—the wealth of talent in the population, the aspirations of its members, and their capacity to arise and become the protagonists of change may all be overlooked. Furthermore, external observers of poverty are all too often unaware of the tendency to allow their own feelings of pity, fear, indignation or ambivalence to affect their reading of society and to base their proposed solutions on the value they place on their own experiences. However, when an effort is participatory, in the sense that it seeks to involve the people themselves in the generation and application of knowledge, as all forge together a path of progress, dualities such as "outsider-insider" and "knowledgeable-ignorant" quickly disappear.

Social Action, Office of Social and Economic Development at the Bahá'í World Centre, 26 November 2012

The Place of Central Appalachia

A community is the mental and spiritual condition of knowing that the place is shared, and that the people who share the place define and limit the possibilities of each other's lives. It is the knowledge that people have of each other, their concern for each other, their trust in each other, the freedom with which they come and go among themselves. (Berry)

Whatever doesn't fit a place is wrong, Berry said. It doesn't matter if it is true or false. If it doesn't belong, it is wrong. Without a standard of place as a measure of real prosperity, Berry said, we will never know what to make of development, technology, research, education, modernization, religion and the environment, or ecosphere. (`http://home2.btconnect.com/tipiglen/berry.html`)

The Place of Central Appalachia

ACCLAIM: NSF Center for Teaching and Learning, Appalachian Collaborative Consortium for Learning, Assesssment, and Instruction in Mathematics. Marshall University, University of Kentucky, University of Louisville, University of Ohio, University of Tennessee, West Virginia University. Mathematics Educators, Mathematicians, and Rural Sociologists.

"ACCLAIM's mission is the cultivation of indigenous leadership capacity for the improvement of school mathematics in rural places. The project aims to (1) understand the rural context as it pertains to learning and teaching mathematics; (2) articulate in scholarly works, including empirical research, the meaning and utility of that learning and teaching among, for, and by rural people; and (3) improve the professional development of mathematics teachers and leaders in and for rural communities."
—`https://sites.google.com/site/acclaimruralmath`

The Place of Central Appalachia

ACCLAIM

Teaching teachers of future teachers: One component of this project was the creation of an interinstitutional doctoral program in mathematics education valuable enough for three cohorts of students to engage in while still paying half tuition and maintaining their full time jobs. Students in the third cohort extend the span of rural places from Alaska to Virginia.

- Developed an effective combination of physical summer courses (initial cohort bonding by fire) and online courses during the academic year.
- Addressed issues in mathematics, mathematics education, and rural sociology.

The Place of Central Appalachia

ACCLAIM

Another was the establishment of dialogue and research on the intersection of mathematics education and rural education.

See in particular "Going Further: A Roadmap to the Works of the ACCLAIM Research Initiative (working paper no. 42, February 2012)" by Howley, available from `http://files.eric.ed.gov/fulltext/ED529588.pdf`.

The Place of Central Appalachia

ACCLAIM

Quantitative research included

- Falsifying rural deficiency in mathematics outcomes
- Providing evidence rather for the effectiveness of math instruction in rural schools
- Disclosing the damage associated with consolidation, both overall and for math outcomes in specific
- Disclosing the advantage of small schools for the math outcomes of impoverished students

The Place of Central Appalachia

ACCLAIM

Qualititative research observed

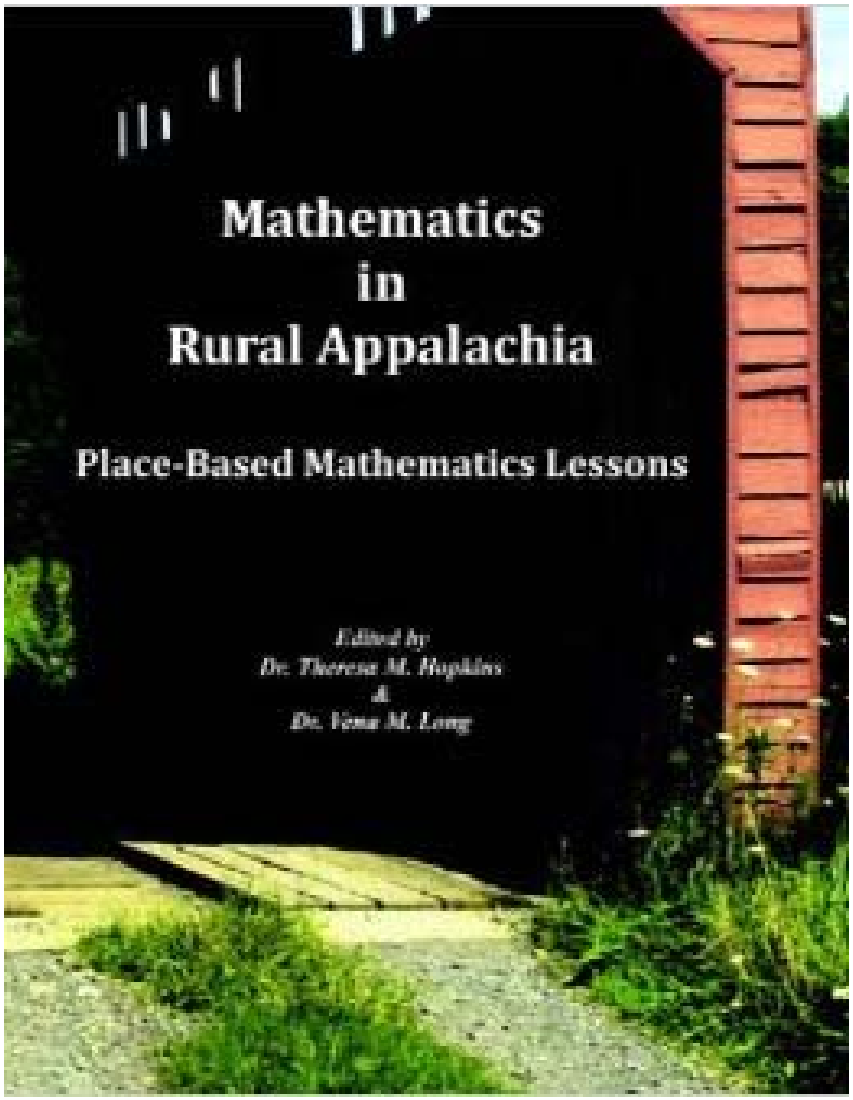
- Rural communities continue to invest their schools with local practical, social, and cultural significance, and this rural difference continues to constitute part of the struggle within rural schooling
- Engagement with improverished students is a significant phase in the struggle to sustain rural community and provision it with decent mathematical educational experience, although egalitarian class structure changes the terms of such engagement
- Structures associated with rural schools (e.g., small size, possibility of focused curriculum, less bureaucracy) seem to facilitate math education reform, but incremental changes do not necessarily entail a shift to instruction vigorously focused on concepts

The Place of Central Appalachia

ACCLAIM

- Part of the difficulty of reform in rural schools may entail a lack of capacity to imagine mathematics as more than a set of useful rules and definitions
- Although placed-based mathematics is quite rare, multiple "existence proofs" suggest that it can thrive given a champion and other auspicious circumstances

The Place of Central Appalachia



The Place of Central Appalachia

AMSP: NSF Math Science Partnership. Eastern Kentucky University, Kentucky State University, Marshall University, Morehead State University, Pikeville College, Union College, University of Kentucky, University of Tennessee, University of Virginia's College at Wise, and about 60 school districts.

A wide range of professional development and course development activities, including a focus on enhancement projects generated by groups of partners based on local concerns.

Funded on the heels of the Appalachian Rural Systemic Initiative

The Place of Central Appalachia

AMSP

- Early teaching experiences for high school students
- Dual credit online courses for high school students
- Early teaching experiences for college students (not necessarily yet majoring in education)
- Revisions of courses for math and science education majors
- Professional development for practicing teachers
- Development of local coordinators and mentors
- Partnership enhancement programs generated from local needs and concerns
- Development of cadre of math coaches

The Place of Central Appalachia

AMSP

NSF Funded RETA Grant to evaluate AMSP undertook an extensive analysis concluding, in part, that though no one AMSP activity was correlated in a statistically significant way to improvements in students' math and science test scores, yet nonetheless, overall participation of a county in AMSP did show statistically significant correlation, even after controlling for such factors as current student score trends and participation in ARSI.

The Place of Central Appalachia

Increase understanding of the place of mathematics, and the mathematics of place

The Place of Mathematical Work

What is the "ZPD" of your place, whether it be an institution, a school, or a community? What are the implications for mathematics instruction and professional development?

The Place of Mathematical Work

Work with partners in a spirit of humble partnership. There are things that you know, and there are things that others know. If you do not approach teaching, professional development, or community capacity building with a sense of superiority, but rather with a sense of authentic collaboration, you will dramatically increase the efficacy of your work together.

Thank you!

Photos

Sihler: http://www.sammlungen.hu-berlin.de/dokumente/12594
Strube: http://msa.maryland.gov/msa/speccol/sc4600/sc4680/html/mackall.html
MacPhail: Personal photo
Gardner: http://vicskeptics.wordpress.com/2010/05/25/vale-martin-gardner
Polyá: http://ecx.images-amazon.com/images/I/51R01GAYEFL.jpg
Vygotsky: https://www.marxists.org/archive/vygotsky/images/portrait.jpg
Dweck: http://ww4.hdnux.com/photos/06/77/71/1834579/5/628x471.jpg
Germain: http://img1.imagesbn.com/p/9781599350622_p0_v1_s260x420.jpg
Ramanujan: http://www.thisiskumbakonam.com/images/photos/image02.jpg
Pine Mountain: Personal photo
Berry: