Percent and Percent Change

Beth Kirby and Carl Lee

University of Kentucky
MA 111

Fall 2009

Course Information

Supplementary Material: *Case Studies for Quantitative Reasoning*, by Madison, Boersma, Diefenderfer, and Dingman.

Course Website:
http://www.ms.uky.edu/~lee/ma111fa09/ma111fa09.html

Percent

Percent

Percent means “per 100”.

Percent

Percent means “per 100”.

So 32 percent or 32% means “32 per 100” or “32 out of 100” or $\frac{32}{100}$ or 0.32.

Percent

Percent means "per 100".

So 32 percent or 32% means "32 per 100" or "32 out of 100" or $\frac{32}{100}$ or 0.32.

What does 25% mean?

Percent

Percent means "per 100".

So 32 percent or 32% means "32 per 100" or "32 out of 100" or $\frac{32}{100}$ or 0.32.

What does 25% mean? $\frac{25}{100}$ or 0.25.

Percent

Percent means "per 100".

So 32 percent or 32% means "32 per 100" or "32 out of 100" or $\frac{32}{100}$ or 0.32.

What does 25% mean? $\frac{25}{100}$ or 0.25.

What does 2% mean?

Percent

Percent means "per 100".

So 32 percent or 32% means "32 per 100" or "32 out of 100" or $\frac{32}{100}$ or 0.32.

What does 25% mean? $\frac{25}{100}$ or 0.25.

What does 2% mean? $\frac{2}{100}$ or 0.02.

Percent

Percent means "per 100".

So 32 percent or 32% means "32 per 100" or "32 out of 100" or $\frac{32}{100}$ or 0.32.

What does 25% mean? $\frac{25}{100}$ or 0.25.

What does 2% mean? $\frac{2}{100}$ or 0.02.

What does 325% mean?

Percent

Percent means "per 100".

So 32 percent or 32% means "32 per 100" or "32 out of 100" or $\frac{32}{100}$ or 0.32.

What does 25% mean? $\frac{25}{100}$ or 0.25.

What does 2% mean? $\frac{2}{100}$ or 0.02.

What does 325% mean? $\frac{325}{100}$ or 3.25.

Percent

Percent means “per 100”.

So 32 percent or 32% means “32 per 100” or “32 out of 100” or $\frac{32}{100}$ or 0.32.

What does 25% mean? $\frac{25}{100}$ or 0.25.

What does 2% mean? $\frac{2}{100}$ or 0.02.

What does 325% mean? $\frac{325}{100}$ or 3.25.

What does 0.07% mean?

Percent

Percent means “per 100”.

So 32 percent or 32% means “32 per 100” or “32 out of 100” or $\frac{32}{100}$ or 0.32.

What does 25% mean? $\frac{25}{100}$ or 0.25.

What does 2% mean? $\frac{2}{100}$ or 0.02.

What does 325% mean? $\frac{325}{100}$ or 3.25.

What does 0.07% mean? $\frac{0.07}{100}$ or 0.0007.

Percent

"32% of 545 is 174.4" means $\frac{32}{100} \times 545 = 174.4$.

Percent

"32% of 545 is 174.4" means $\frac{32}{100} \times 545 = 174.4$.
It also means $\frac{32}{100} = \frac{174.4}{545}$.

Percent

"32% of 545 is 174.4" means $\frac{32}{100} \times 545 = 174.4$.
It also means $\frac{32}{100} = \frac{174.4}{545}$.

"P% of N is A" means $\frac{P}{100} \times N = A$.

Percent

"32% of 545 is 174.4" means $\frac{32}{100} \times 545 = 174.4$.
It also means $\frac{32}{100} = \frac{174.4}{545}$.

"P% of N is A" means $\frac{P}{100} \times N = A$.
It also means $\frac{P}{100} = \frac{A}{N}$.

Percent

"32% of 545 is 174.4" means $\frac{32}{100} \times 545 = 174.4$.
It also means $\frac{32}{100} = \frac{174.4}{545}$.

"P% of N is A" means $\frac{P}{100} \times N = A$.
It also means $\frac{P}{100} = \frac{A}{N}$.

In the above example, 545 is the base for the percent. The base for a percent is the quantity to which the percent applies.

Percent

"32% of 545 is 174.4" means $\frac{32}{100} \times 545 = 174.4$.
It also means $\frac{32}{100} = \frac{174.4}{545}$.

"P% of N is A" means $\frac{P}{100} \times N = A$.
It also means $\frac{P}{100} = \frac{A}{N}$.

In the above example, 545 is the base for the percent. The base for a percent is the quantity to which the percent applies.

A very common error is to use the incorrect base for a percent.

Given P and N, Find A

If a county proposes charging a tax of 0.5% on a \$270 purchase, what is the amount of tax paid?

Given P and N, Find A

If a county proposes charging a tax of 0.5% on a \$270 purchase, what is the amount of tax paid?

$$\frac{0.5}{100} = \frac{A}{270}$$

so

$$A = \frac{0.5}{100} \times 270 = \$1.35.$$

Given P and A, Find N

Jason was transfered to a different school and now must drive 48 miles to school, which is 225% of the previous distance. What was the previous distance?

Given P and A, Find N

Jason was transfered to a different school and now must drive 48 miles to school, which is 225% of the previous distance. What was the previous distance?

$$\frac{225}{100} = \frac{48}{N}$$

so

$$225 \times N = 100 \times 48,$$

or

$$N = \frac{100 \times 48}{225} \approx 21.3 \text{ miles.}$$

Given N and A, find P

Suppose that 25 students enrolled in MA 111 last year and 75 enrolled this year. What percentage of 25 is 75?

Given N and A, find P

Suppose that 25 students enrolled in MA 111 last year and 75 enrolled this year. What percentage of 25 is 75?

$$\frac{P}{100} = \frac{75}{25}$$

so

$$P = \frac{75}{25} \times 100 = 300.$$

Thus the number of students enrolled this year is 300% of the number enrolled last year.

Other Terms

One mill is \$1 per \$1000. Taxes on property are often stated in terms of mills.

Other Terms

One mill is $1 per $1000. Taxes on property are often stated in terms of mills.

If the tax rate is 6 mills, what is the rate expressed as a percent?

Other Terms

One mill is \$1 per \$1000. Taxes on property are often stated in terms of mills.

If the tax rate is 6 mills, what is the rate expressed as a percent?

$$\frac{P}{100} = \frac{6}{1000}$$

so

$$P = \frac{6}{1000} \times 100 = 0.6.$$

So the rate is 0.6%, meaning \$0.60 per \$100.

Changes Measured by Percents

If the population of a town was 1000 and it increased by 25%, what is the new population?

If a pair of shoes cost $60 and the price was reduced 30%, what is the new price?

Changes Measured by Percents

If the population of a town was 1000 and it increased by 25%, what is the new population?

Changes Measured by Percents

If the population of a town was 1000 and it increased by 25%, what is the new population?

Method 1: Find 25% of 1000 and add it to 1000.

Changes Measured by Percents

If the population of a town was 1000 and it increased by 25%, what is the new population?

Method 1: Find 25% of 1000 and add it to 1000.
$\frac{25}{100} \times 1000 = 250$. So the new population is
$1000 + 250 = 1250$.

Changes Measured by Percents

If the population of a town was 1000 and it increased by 25%, what is the new population?

Method 1: Find 25% of 1000 and add it to 1000.
$\frac{25}{100} \times 1000 = 250$. So the new population is
$1000 + 250 = 1250$.

Note that this is equivalent to

$$1000 + \frac{25}{100} \times 1000$$

or

$$1000\left(1 + \frac{25}{100}\right).$$

Changes Measured by Percents

So we have Method 2: Multiply 1000 by $(1 + \frac{25}{100})$:

$$1000 \times \left(1 + \frac{25}{100}\right) = 1250.$$

Changes Measured by Percents

If a pair of shoes cost \$60 and the price was reduced 30%, what is the new price?

Changes Measured by Percents

If a pair of shoes cost \$60 and the price was reduced 30%, what is the new price?

Method 1: Find 30% of \$60 and subtract it from \$60.

Changes Measured by Percents

If a pair of shoes cost \$60 and the price was reduced 30%, what is the new price?

Method 1: Find 30% of \$60 and subtract it from \$60.
$\frac{30}{100} \times 60 = 18$. So the new price is $60 - 18 = \$42$.

Changes Measured by Percents

If a pair of shoes cost \$60 and the price was reduced 30%, what is the new price?

Method 1: Find 30% of \$60 and subtract it from \$60.
$\frac{30}{100} \times 60 = 18$. So the new price is $60 - 18 = \$42$.

Note that this is equivalent to

$$60 - \frac{30}{100} \times 60$$

or

$$60\left(1 - \frac{30}{100}\right).$$

Changes Measured by Percents

So we have Method 2: Multiply 60 by $(1 - \frac{30}{100})$:

$$60 \times \left(1 - \frac{30}{100}\right) = 42.$$

Changes Measured by Percents

General Situation:

A is increased by $P\%$ of A to get B:

$$B = A\left(1 + \frac{P}{100}\right).$$

Changes Measured by Percents

General Situation:

A is increased by $P\%$ of A to get B:

$$B = A\left(1 + \frac{P}{100}\right).$$

A is decreased by $P\%$ of A to get B:

$$B = A\left(1 - \frac{P}{100}\right).$$

Given A and P, find B

If 75000 is increased by 20%, what is the result?

Given A and P, find B

If 75000 is increased by 20%, what is the result?

$$B = 75000\left(1 + \frac{20}{100}\right) = 90000.$$

Given A and P, find B

If 75000 is decreased by 30%, what is the result?

Given A and P, find B

If 75000 is decreased by 30%, what is the result?

$$B = 75000\left(1 - \frac{30}{100}\right) = 52500.$$

Given A and P, find B

If 80 is increased by 20% and then the result is decreased by 25%, what is the final result?

Given A and P, find B

If 80 is increased by 20% and then the result is decreased by 25%, what is the final result?

Two steps: First...

$$80\left(1+\frac{20}{100}\right)=96.$$

Given A and P, find B

If 80 is increased by 20% and then the result is decreased by 25%, what is the final result?

Two steps: First...

$$80\left(1+\frac{20}{100}\right)=96.$$

Use this result...

$$96\left(1-\frac{25}{100}\right)=72.$$

Thus, the final result is 72.

Given A and P, find B

If 80 is decreased by 5%, what is the result?

Given A and P, find B

If 80 is decreased by 5%, what is the result?

$$80\left(1-\frac{5}{100}\right)=80(0.95)=76.$$

Given A and P, find B

If 80 is decreased by 5%, what is the result?

$$80\left(1-\frac{5}{100}\right)=80(0.95)=76.$$

The process of increasing by 20% and then decreasing by 25% is *not equivalent* to decreasing by 5% because the base of the percent changes.

Given A and B, find P

If

$$A\left(1+\frac{P}{100}\right)=B$$

then

$$\frac{P}{100}=\frac{B}{A}-1$$

or

$$P=\frac{B-A}{A}\times 100\%.$$

Given A and B, Find P

If the cost of a gallon of gasoline increases from \$1.70 to \$1.90, what is the percent increase?

Given A and B, Find P

If the cost of a gallon of gasoline increases from \$1.70 to \$1.90, what is the percent increase?

$$\frac{\$1.90 - \$1.70}{\$1.70} \times 100\% \approx 11.76\%.$$

Given A and B, Find P

If the cost of a gallon of gasoline decreases from $1.90 to $1.70, what is the percent change?

Given A and B, Find P

If the cost of a gallon of gasoline decreases from \$1.90 to \$1.70, what is the percent change?

$$\frac{\$1.70 - \$1.90}{\$1.90} \times 100\% \approx -10.53\%.$$

Given A and B, Find P

By what percent is 40 increased by to get 50?

Given A and B, Find P

By what percent is 40 increased by to get 50?

$$\frac{50-40}{40} \times 100\% = 25\%.$$

Given B and P, Find A

If the population of a city in 2008 was 75,870 and this was an increase of 6% since 1998, what was the population in 1998? Note that the base A of the percent is not known.

Given B and P, Find A

If the population of a city in 2008 was 75,870 and this was an increase of 6% since 1998, what was the population in 1998? Note that the base A of the percent is not known.

$$B = A\left(1 + \frac{P}{100}\right)$$

Given B and P, Find A

If the population of a city in 2008 was 75,870 and this was an increase of 6% since 1998, what was the population in 1998? Note that the base A of the percent is not known.

$$B = A\left(1 + \frac{P}{100}\right)$$

$$75870 = A\left(1 + \frac{6}{100}\right)$$

Given B and P, Find A

If the population of a city in 2008 was 75,870 and this was an increase of 6% since 1998, what was the population in 1998? Note that the base A of the percent is not known.

$$B = A\left(1 + \frac{P}{100}\right)$$

$$75870 = A\left(1 + \frac{6}{100}\right)$$

$$75870 = A(1.06)$$

Given B and P, Find A

If the population of a city in 2008 was 75,870 and this was an increase of 6% since 1998, what was the population in 1998? Note that the base A of the percent is not known.

$$B = A\left(1 + \frac{P}{100}\right)$$

$$75870 = A\left(1 + \frac{6}{100}\right)$$

$$75870 = A(1.06)$$

$$A = \frac{75870}{1.06} = 71575.$$

Given B and P, Find A

If a laptop computer sells for $1100 in 2009 and this is a decrease of 7% in the price since 2007, what was the price in 2007?

Given B and P, Find A

If a laptop computer sells for $1100 in 2009 and this is a decrease of 7% in the price since 2007, what was the price in 2007?

$$B = A\left(1 - \frac{P}{100}\right)$$

Given B and P, Find A

If a laptop computer sells for $1100 in 2009 and this is a decrease of 7% in the price since 2007, what was the price in 2007?

$$B = A\left(1 - \frac{P}{100}\right)$$

$$1100 = A\left(1 - \frac{7}{100}\right)$$

Given B and P, Find A

If a laptop computer sells for $1100 in 2009 and this is a decrease of 7% in the price since 2007, what was the price in 2007?

$$B = A\left(1 - \frac{P}{100}\right)$$

$$1100 = A\left(1 - \frac{7}{100}\right)$$

$$1100 = A(0.93)$$

Given B and P, Find A

If a laptop computer sells for $1100 in 2009 and this is a decrease of 7% in the price since 2007, what was the price in 2007?

$$B = A\left(1 - \frac{P}{100}\right)$$

$$1100 = A\left(1 - \frac{7}{100}\right)$$

$$1100 = A(0.93)$$

$$A = \frac{1100}{0.93} \approx \$1183.$$

A Note on Accuracy

Any example prefaced with an asterisk (*) uses made-up data.

Absolute vs. Relative Change in Percent

Example: In October 2008, the United States Congress had a 15% approval rating. In January 2009, Congress had a 40% approval rating.

The absolute increase in the percentage of Americans who approve of Congress is 40% - 15% = 25%.

This is often called an increase in percentage points. We would say that there was an increase of 25 percentage points.

Absolute vs. Relative Change

Example: In October 2008, the United States Congress had a 15% approval rating. In January 2009, Congress had a 40% approval rating.

The relative increase uses the initial percentage (15%) as the base for the calculation:

$$\frac{40\% - 15\%}{15\%} \approx 1.667 = 166.7\%.$$

This is sometimes called a percent increase.
Compared to the October 2008 approval ratings, the approval rating increased by 166% in January 2009.

Tips for Distinguishing Absolute and Relative Change

- An absolute change is often worded as, "The change in the percentage of..."
- A relative change is often worded as, "The percent change in the percentage of..."

Relative Change

*Example: If the Math Department had 60 graduate students in 2006 and 66 graduate students in 2008, what was the percent increase in the number of graduate students?

Relative Change

*Example: If the Math Department had 60 graduate students in 2006 and 66 graduate students in 2008, what was the percent increase in the number of graduate students?

$$\frac{66-60}{60} = \frac{6}{60} = 0.1 = 10\%.$$

There was a 10% increase in the number of graduate students in the Math Department.

Relative Change

*Example: In 2006, 30% of the graduate students in the Math Department were women. In 2008, 45% of the graduate students in the Math Department were women. What was the percent increase in the percentage of graduate students who are women?

Relative Change

*Example: In 2006, 30% of the graduate students in the Math Department were women. In 2008, 45% of the graduate students in the Math Department were women. What was the percent increase in the percentage of graduate students who are women?

$$\frac{45\% - 30\%}{30\%} = \frac{15\%}{30\%} = 0.5 = 50\%.$$

There was a 50% increase in the percentage of graduate students who are women.

Relative Change

*Example: In 2006, 30% of the graduate students in the Math Department were women. In 2008, 45% of the graduate students in the Math Department were women. What was the percent increase in the percentage of graduate students who are women?

$$\frac{45\% - 30\%}{30\%} = \frac{15\%}{30\%} = 0.5 = 50\%.$$

There was a 50% increase in the percentage of graduate students who are women.

Notice that the form of the calculation is very similar to that of the previous example.

Example

In 1972, 46% of college-age Americans read a newspaper every day. Today it's only 21% percent.

What is the (absolute) change in the percentage?
What is the (relative) percent change in the percentage?

Example

In 1972, 46% of college-age Americans read a newspaper every day. Today it's only 21% percent.

What is the (absolute) change in the percentage?
What is the (relative) percent change in the percentage?

The absolute change in the percentage is 21%-46% = -25%.

Example

In 1972, 46% of college-age Americans read a newspaper every day. Today it's only 21% percent.

What is the (absolute) change in the percentage?
What is the (relative) percent change in the percentage?

The absolute change in the percentage is 21%-46% = -25%.

The relative change in the percentage is

$$\frac{21\% - 46\%}{46\%} = \frac{-25\%}{46\%} \approx -0.54 = -54\%.$$

Example

If the risk of contracting a disease rose from 0.2% to 0.28%, find the change in percentage points. Then find the relative change of the percentage.

Example

If the risk of contracting a disease rose from 0.2% to 0.28%, find the change in percentage points. Then find the relative change of the percentage.

The absolute change in the percentage is 0.28%-0.2% = 0.08%, or 0.08 percentage points.

The relative change is

$$\frac{0.28\% - 0.2\%}{0.2\%} = \frac{0.08\%}{0.2\%} = 0.4 = 40\%.$$

Case Study 1: Tax Rates

1. Create an organized list for how the tax rates for \$5,000 taxes on \$30,000 income and \$53,000 taxes on \$200,000 income are stated in the seven letters under consideration.

2. Which of the stated tax rates are correct and which are incorrect?

3. What is the mistake that Mr. Massey probably made in computing the tax rates?

4. Which of the six letters responding to Mr. Massey have errors, and what are those errors?

Case Study 1: Tax Rates

5. Which, if any, of the letters dispute the amounts of tax cited by Mr. Massey: $5,000 on $30,000 and $53,000 on $200,000?

6. For each of the six responses to Mr. Massey's letter, write 2-3 sentences critiquing the letter as to its accuracy, tone, and effectiveness.

Case Study 2: Other People's Money

1. Find and describe the dollar amounts, percents, and percent changes in the article.
2. Describe each of the dollar amounts in terms of something familiar enough so that you have a better understanding of the size of the dollar amount. For example, $600,000 buys 20 new automobiles.
3. From the information in the article, find the amount of the annual federal budget.
4. From the information in the article, find the number of people employed in this country.

Case Study 2: Other People's Money

5. From the information in the article, find the number of unemployed in this country.
6. Check the work after the sentence on page 2, "Let's do the math here." Compare your results to those stated by the author.

Case Study 3: Big Stink in Little Elkins

1. Compare the two following statements from the article. Are they consistent?
   1.1 The Elkins City Council voted in 1998 to reduce sewer rates by 15%.
   1.2 Pruett told Lisle that the sewer bills were supposed to be running 85% of what the water bills are.

2. Compare the two following statements from the article. Are they consistent?
   2.1 But when Pruett put a pencil to what he had actually been paying, it amounted to 89 percent of the monthly water bill.
   2.2 Sewer users have been overpaying by 4 percent each month for three years.

Case Study 3: Big Stink in Little Elkins

3. Use the information in the article to find how much the City of Elkins received from residential customers in total sewer payments during the three-year period following the 1998 reduction.

4. Consider a different version of the Elkins sewer cost reduction. Suppose that Elkins' sewer rates were 60% of one's water bill and the city decided to lower that to 45% of the water bill. Express this change in a reduction of percentage points as a relative reduction.

Case Study 4: Trainees Fueling Agency's Optimism

1. Using the data in the article, find the percent increase in the number of trainees from August 31 to September 30 and from September 30 to October 31.

2. What are the percents 65%, 76%, 80% that are written on the bars in the graphic?

3. The text that accompanies the graphic includeds: "Caseworker trainees increasing. The number of Division of Children and Family Services caseworker trainees increased by 15% from August to October." Do the given data support these two statements? If not, give the correct conclusions that are supported by the data provided.

Case Study 4: Trainees Fueling Agency's Optimism

4. Other increases that are relevant in discussing the topic of this article are the increases in the total positions, whether filled or not. Find these three percent increases.

5. Critique the graphic in the article. Is the graph effective in conveying the information? If so, tell why, and if not, describe a graphic that would convey the information better.

Case Study 5:

1.
   I. More mothers of babies under 1 are staying home.
   II. After a quarter-century in which women with young children poured into the workplace, last year brought the first decline in the percentage of women who have babies younger than 1 year old and are in the work force.
   III. A new Census Bureau report said 55 percent of women with infants were in the labor force in June 2000, compared with 59 percent two years earlier.
   IV. Fewer mothers of infants at work.
   V. The percentage of working mothers with babies decreased in 2000.
   VI. Of the mothers in the work force who had infants under 1, 34 percent worked full time, and 17 percent part time, the report said. Four percent were unemployed but wanted to work.

Case Study 5:

2. What are the relationships of the six assertions? Which assertions say essentially the same thing? Which assertions differ?

3. Do you find other assertions that are part of the main quantitative message? If so what are they and how do they relate to the six above?

Case Study 5:

4. The first paragraph of the online version of this article is different from the print version. The online version read (see II above): *After a quarter-century in which women with young children poured into the workplace, last year brought the first decline in the percentage of women who have babies younger than 1 year old and are in the work force.* The print version said: *After a quarter-century in which women with young children poured into the workplace, the percentage of women in the labor force who had babies younger than 1-year old declined last year.* Compare and contrast these first two paragraphs.
5. Which one of the above assertions appears to be the correct message of the article?

Case Study 5:

6. Do assertions I and IV follow from assertion III?
7. Does assertion II follow from assertion III?
8. In assertion VI, what is the base of the 34 percent, 17 percent, and 4 percent?
9. How should assertion VI be stated so that it is correct?