

## Credit Cards

Credit cards often charge very high interest rates, and extra fees and penalties occur when the outstanding balance is not paid off in full. On the other hand, when used responsibly, credit cards can act as short-term interest-free loans, and on top of that, you may even earn rewards while using the card. (Thus, if *properly* a credit cards can be a very good thing)

We will look at some of the mathematics involved in credit card billing. We will why it is so easy to get deep into debt by misusing credit cards.

### Example 1:

Suppose you have a brand new credit card which charges 18% APR (*annual percentage rate*). Most credit cards use monthly billing, and we will assume this card does the same. The *periodic interest rate* is then

$$\frac{\text{Annual percentage rate}}{\text{Number billing periods per year}} = \frac{18\%}{12} = 1.5\%$$

(Be weary of cards that claim very low interest rates. They may be advertising their monthly rate, not their yearly rate!)

Suppose you charge \$742.36 on the card on April 2, 2011 and you charge an additional \$451.24 on April 10, 2011, and you make no other transactions on the card.

Below is a summary of what you may see in the top right corner of the bill.

Previous Balance	0.00
Plus Purchases	742.36
Plus Fees/Charges	0.00
Less Payments	0.00
Interest Rate	1.5%
Total Amount Due	742.36
Minimum Amount Due	20.00
Statement Date	April 6, 2011
Payment Due Date	April 26, 2011

A few things to notice:

- (a) You put  $742.36 + 451.24 = \$1193.60$  on the card but only  $\$742.36$  appears on the bill. What happened to the  $\$451.24$ ? Notice the statement date is April 6, which means transactions posted after April 6 will appear on a later bill. Thus, the  $\$742.36$  charge appears on this bill, but the  $\$451.24$  will appear on the next bill.
- (b) The interest rate is stated in monthly terms, not annual terms.
- (c) Notice the interest has not taken affect yet. You spent  $\$742.36$  and you owe  $\$742.36$ . For the moment, this is an interest free loan!
- (d) The time difference between the statement date and the due date is referred to as the *grace period*. Thus, this card has a 20 day grace period.

We will consider two scenarios.

- First Scenario: You pay back the total amount due each time you receive a billing statement.
- Second Scenario: You only pay the minimum amount due each time you receive a billing statement.

**First Scenario** You decide to pay off the balance in full on the first bill. Below is the top of the next month's billing statement.

Previous Balance	742.36
Plus Purchases	451.24
Plus Fees/Charges	0.00
Less Payments	742.36
Interest Rate	1.5%
Total Amount Due	451.24
Minimum Amount Due	20.00
Statement Date	May 6, 2011
Payment Due Date	May 26, 2011

The *previous balance* is the total amount you owed last period. The amount you paid back last period appears in the line *less payments*. The amount in the *Plus Fees/Charges* line is \$0.00 since you paid off the entire balance last period (i.e., since the *previous balance* and *less payments* lines are equal). Later examples will include cases with nonzero fees and charges.

Let's suppose you decide to pay off the balance in full again. Your next billing statement would then look like this:

Previous Balance	451.24
Plus Purchases	0.00
Plus Fees/Charges	0.00
Less Payments	451.24
Interest Rate	1.5%
Total Amount Due	0.00
Minimum Amount Due	0.00
Statement Date	June 6, 2011
Payment Due Date	June 26, 2011

You made no new purchases, and since you paid everything off in full last period, there are no fees or charges.

Thus, in this first scenario you essentially received two interest free loans which you paid back in a very short period of time.

It's easy to understand your credit card billing statements if you always pay the *total amount due*. Interpreting these statements can be more difficult if you don't always pay the total amount due.

**Second Scenario** In this scenario, you are tempted by the minimum amount due. (Why pay \$742.36 if you are only required to pay \$20.00, right?) The top of the next month's billing statement is shown below.

Previous Balance	742.36
Plus Purchases	451.24
Plus Fees/Charges	17.60
Less Payments	20.00
Interest Rate	1.5%
Total Amount Due	1191.20
Minimum Amount Due	20.00
Statement Date	May 6, 2011
Payment Due Date	May 26, 2011

The \$20 you paid on the April bill is reflected in the *less payments* line. The total amount due is obtained by adding the *previous balance*, *purchases*, and *fees/charges*, and subtracting the *previous payment*:

$$\begin{aligned} \text{total amount due} &= \text{previous balance} + \text{purchases} + \text{fees/charges} - \text{payments} \\ &= 742.36 + 451.24 + 17.60 - 20.00 = 1191.20 \end{aligned}$$

The \$742.36, \$451.24 and \$20.00 are self-explanatory, but where did the \$17.60 come from? Since you did not pay the total amount due last month, the interest rate will now be applied to the *balance subject to finance charges*, which is the previous balance plus the new purchases.

$$\begin{aligned} \text{balance subject to finance charges} &= \text{previous balance} + \text{purchases} - \text{payments} \\ &= 742.36 + 451.24 - 20.00 = 1173.60 \end{aligned}$$

Then

$$\text{Fees/Charges} = 1173.60 * (1.5\%) = 1173.60 * \frac{1.5}{100} = 17.60,$$

that is, the *fees/charges* line is obtained by applying the *periodic interest rate* to the *balance subject to finance charges*.

Notice that the interest is being charged not only on the left over balance from the first billing cycle ( $742.36 - 20 = \$722.36$ ), but it is also charged on the new purchases ( $\$451.24$ ).

Now, notice the *fee/charge* adds  $\$17.60$  to the *total amount due*, but last month you only paid  $\$20.00$  toward the balance. The net effect is that only  $20.00 - 17.60 = \$2.40$  has been applied to the balance! (Only 12% of your  $\$20.00$  payment went towards the balance, and 88% of your  $\$20.00$  payment went towards fees.)

Suppose we pay the minimum of  $\$20$  again, and make no additional charges on the card. The June billing statement is summarized below.

Previous Balance	1191.20
Plus Purchases	0.00
Plus Fees/Charges	17.57
Less Payments	20.00
Interest Rate	1.5%
Total Amount Due	1188.77
Minimum Amount Due	20.00
Statement Date	June 6, 2011
Payment Due Date	June 26, 2011

As before,

$$\begin{aligned} \text{balance subject to finance charges} &= \text{previous balance} + \text{purchases} - \text{payments} \\ &= 1191.20 + 0.00 - 20.00 = 1171.20 \end{aligned}$$

Then

$$\text{Fees/Charges} = 1171.20 * (1.5\%) = 1171.20 * \frac{1.5}{100} = 17.57$$

and

$$\text{Total amount due} = 1191.20 + 0.00 + 17.57 - 20.00 = 1188.77$$

Summarizing, you have borrowed \$1193.60, you paid  $20 + 20 = \$40$  towards that balance, and have incurred  $17.60 + 17.57 = \$35.17$  in fees, which means effectively only  $40 - 35.18 = \$4.83$  of the \$40 you paid has gone towards paying back the balance!

Try filling out the next month's statement.

Previous Balance	
Plus Purchases	0.00
Plus Fees/Charges	
Less Payments	20.00
Interest Rate	1.5%
Total Amount Due	
Minimum Amount Due	20.00
Statement Date	July 6, 2011
Payment Due Date	July 26, 2011

Incidentally, if you keep going at this rate, it will take you 145 months (over 12 years) to pay off the debt, and you will end up paying back over \$2870 dollars whereas you borrowed \$1193.60. (The cost to borrow the the \$1193.60 in this case is  $2870 - 1193.60 = \$1676.40!$ )

Needless to say, paying back only the minimum due is typically a very bad idea!

Notice the interest rate does NOT apply until an outstanding balance is left unpaid, thus, IF you pay the total amount due every time, the card acts like an interest free loan. However, if an outstanding balance is left unpaid, all further transactions are subject to the interest rate until the total amount due is paid off. The next example shows illustrates this in an extreme case.

### **Example 2**

Suppose you make two purchases again, say on April 2 and May 5. On April 2 you charge \$35.00 on your card, and on May 5 you charge \$1000.00. This time let's assume the annual percentage rate is 24%. Remember, on the statement this will show up as 2% since the statement shows the monthly interest, not the annual percentage rate.

The first billing statement is shown below.

Previous Balance	0.00
Plus Purchases	35.00
Plus Fees/Charges	0.00
Less Payments	0.00
Interest Rate	2.0%
Total Amount Due	35.00
Minimum Amount Due	30.00
Statement Date	April 6, 2011
Payment Due Date	April 26, 2011

The *fees/charges* is 0.00 since you did not have a previous balance. Thus, if you pay this off in full, you will have borrowed the \$35.00 interest free.

We will consider two scenarios again.

- First Scenario: You pay back the total amount due each time you receive a billing statement.
- Second Scenario: You pay the minimum amount due for the first month and you pay the total balance due for the second month.

**First Scenario** You pay the total amount due. Your next statement is shown below.

Previous Balance	35.00
Plus Purchases	1000.00
Plus Fees/Charges	0.00
Less Payments	35.00
Interest Rate	2.0%
Total Amount Due	1000.00
Minimum Amount Due	30.00
Statement Date	May 6, 2011
Payment Due Date	May 26, 2011

The fees/charges is 0.00 since you paid the total amount due last month. Suppose you pay this months total amount due. The next billing statement is shown below.

Previous Balance	1000.00
Plus Purchases	0.00
Plus Fees/Charges	0.00
Less Payments	1000.00
Interest Rate	2.0%
Total Amount Due	0.00
Minimum Amount Due	0.00
Statement Date	June 6, 2011
Payment Due Date	June 26, 2011

You made no new purchases, and since you paid everything off in full last period, there are no fees or charges. In this scenario, you managed to borrow \$1035.00 interest free.

**Second Scenario** Suppose instead you opted to pay the minimum balance on the first month's statement. The next statement is shown below.



Previous Balance	35.00
Plus Purchases	1000.00
Plus Fees/Charges	20.10
Less Payments	30.00
Interest Rate	2.0%
Total Amount Due	1025.10
Minimum Amount Due	30.00
Statement Date	May 6, 2011
Payment Due Date	May 26, 2011

balance subject to finance charges =  $35.00 + 1000.00 - 30.00 = 1005.00$

Then

$$\text{Fees/Charges} = 1005.00 * (2.0\%) = 1005.00 * \frac{2.0}{100} = 20.10$$

and

$$\text{Total amount due} = 35.00 + 1000.00 + 20.10 - 30.00 = 1025.10$$

Suppose we pay the total amount due. Over the two months, we borrowed  $\$35 + \$1000 = \$1035.00$  and we had to pay back  $30 + 1025.10 = 1055.10$ . Thus, the cost of the loan was  $1055.10 - 1035.00 = \$20.10$

Look at the difference between these two scenarios very closely. Leaving \$5.00 unpaid on the first month's bill cost us \$20.10 on the second month's bill! Had we paid off the first month's bill in full, we would not have been charged any fees on the \$1000.00 during May's billing cycle.

In real life, credit cards can actually be more complicated than this. For instance, different types of purchases may be subject to different interest rates. Also, we have assumed our borrower has made at least the minimum payment each month. If a payment is late or missed, additional fees may be applied, the card holder's credit line may be reduced (the credit line is the amount the credit card company is willing to lend the card holder),

the interest rate may increase, and the card holder may have to pay at this much higher rate for a long time before they are given back their original rate.