

Chapter 1

Notes for Instructors

Content

The first chapter of Long and DeTemple provides a foundation for the rest of the book. It is important that you cover this chapter, especially if you plan to require your students to read the textbook. I say this, in part, because Pólya's Problem Solving Principles are covered in Section 1.3. Every example problem in the textbook follows Pólya's problems solving methodology. Although it is important to cover Chapter 1, it should be noted that it is not hard to imagine how one could spend months on the problems in Chapter 1. I devoted 3 weeks to chapter 1, but, in retrospect, I think it would have been better to spend only two weeks on this chapter..

There are many problems in Chapter 1 which could be used in an elementary classroom. This helps to provide some motivation for the course. There are also more sophisticated problems. It should be noted that some students may find the more sophisticated problems easier than those which appear to be more appropriate for an elementary classroom. College students have been trained to use variables whenever possible. Many problems can be solved without using variables. For example, elementary students will often attempt to solve problems by working examples. *Using a variable* is one of the problem solving strategies discussed in Chapter 1, but there are many others which college students will need to recall.

Notes and Suggestions

Since you will not have the luxury of spending months on Chapter 1, I want to highlight some important ideas and topics in Chapter 1 that will be useful to the students later in the course.

- **Pólya's Problem Solving Principles:** This approach is used to solve problems throughout the textbook. It is particularly important that you highlight the "Look Back" step, a step which is often ignored.
- **Pascal's Triangle:** This triangle is important because it can be used in the chapter on probability to evaluate combinations.
- **Gauss's Trick:** I like this trick. I suppose that one could teach this course without discussing Gauss's Trick, but I have found problems later in the text for which this trick comes in handy. Moreover, I think there are some good ideas which come along with problems that require Gauss's trick. First, Gauss's trick uses a variable in a clever way. Second, students will be required to add equations. (This does appear to be a novel idea to some of them.) Finally, I like to use Gauss's trick to find the sum of the first n terms in an arithmetic progression because these problems have several layers. Students must begin by finding a formula for the n^{th} term of the arithmetic progression. They must also understand how to get the formula for the $(n - 1)^{st}$ term. That is, students can often understand why $1 + 2 + \dots + 100 = 100 + 99 + \dots + 1$, but it is more difficult to see that $1 + 3 + \dots + (2n - 1) = (2n - 1) + (2n - 3) + \dots + 1$.

- **Triangular Numbers:** This is another example where you can use Gauss's trick. Moreover, there are several problems in later sections which use the triangular numbers.
- **Work Backward:** It would be possible to teach the course without discussing this strategy at the beginning of the course. If this strategy is needed later, it could be discussed at that time. Nevertheless, I like this strategy. I like to cover this strategy early in the course because you can use problems similar to Example 1.11 on page 50 to teach students how to write good solutions. There are lots of variations on the game of NIM (one example is described in Example 1.11) which you can include on homework assignments. I also like these problems because I think they are easier to solve if you examine similar smaller problems, that is, if you begin with a smaller pile. This employs another problem solving strategy. Also, students are likely to appreciate the value of manipulatives when solving these problems. Many students find that the problem is much easier to solve if they actually have coins or markers with which to work.