Chapter 4
Notes for Instructors

Content

The focus of this chapter is Number Theory. I only had time to cover sections 4.1–4.3 of this chapter. If you have time, it would certainly be nice to cover section 4.4 because it is always nice to give students a taste of the applications. If this course is ever restructured to have a workshop day, I certainly think that section 4.4 could be examined during a workshop day. In the present structure of the course, I found that it was more important to dedicate time to concepts instead of applications that students were likely to forget.

Manipulatives

I did not have time to incorporate many manipulatives into my lessons for this chapter, however the text does mention a few manipulative that certainly would be useful if you have the foresight to use them. For instance, in section 4.1, you could certainly use any of the blocks that we have in the Mathskellar for the array models. There is an activity on page 228 which uses rectangular arrays. Also, we have the Cuisenaire® rods which are used in question 13 of section 4.3.

Notes and Suggestions

Notes on Section 4.1:*Divisibility of Natural Numbers*

- Students often have difficulty with the definition for "divides." For example, since we can evenly divide 6 by 2, some may want to say that 6 divides 2. I think they get really confused if you use the notation for divides. That is to say that they have trouble differentiating between $6/2$ and $2|6$ because the numbers get rearranged.

- I think it is good to point out that the number 1 is NOT prime. I like to ask them if one is prime. Then I have them read the definition and ask the question again. It is important that students read definitions. Too often students believe that they have read the material if their eyes have gone over the words on the page. They are not digesting the material.

- When students construct factor trees, I find that it is helpful for them to circle all the prime numbers in the tree. I have found that some students will not get the correct prime factorization from a factor tree because they forget about primes that are not on the bottom tier of the tree.

- You should direct the students' attention to the Fundamental Theorem of Arithmetic because you will need it to show that that $\sqrt{2}$ is irrational. You may at this point want to use the FTA to argue that if 2 is a factor of a^2, then 2 must be a factor of a. For that matter, if the prime p is a factor of a^2, then p must be factor of a. You might also want to ask them if this is also true when p is not prime. This will give them another opportunity to find a counterexample. These ideas will be necessary when you prove that $\sqrt{2}$ is irrational. It also seems that students need some time to digest these ideas.

Therefore it is good to cover them in this section and to remind them of these ideas when you actually show that $\sqrt{2}$ is irrational in section 7.1.

- I really like the *Prime Divisors of n Theorem* on page 237 of the textbook. I think it is good to mention that this theorem is concerned with efficiency. That is to say, it cuts down on the amount of work that we need to do to determine if a natural number is prime or composite. I like to require that my students use this theorem when they show that a number is prime. I also like note that if n is composite, then there is a short proof (one sentence) that n is composite. You do not need to use this theorem to prove that n is composite. It is really only useful in proving that a number is prime (though it can help you find a factor if n is not prime). I really like questions 11 and 13 in section 4.1 to test students understanding of this theorem.

- I do not know how use the sieve of Eratosthenes is. I think it could be omitted if you are in a crunch for time.

Notes on Section 4.2 *Tests for Divisibility*

- My main complaint with this section is a lack of proofs. For example, we just give them clever tricks for testing divisibility by 3 and 9, but we don't explain them. The tricks are cute, but it goes against what I strive to teach students in this course: You should not just teach algorithms; you should understand them. If you have time to do a little modular arithmetic, there are proofs for these tests. They cover these proofs in Section 8.3 in *Mathematics Beyond the Numbers* by George T. Gilbert and Rhonda L. Hatcher. This is the book that is currently (Spring 2004) being used for MA 111. You should be able to get a copy of the book from Elizabeth or one of the T.A.'s that is teaching MA 111. I had done proofs of several of the divisibility tests for MA 111 when I taught that course. I have attached a handout I gave my MA 111 students which includes many of these proofs.

- There are some decent problems at the end of section 8.3 in *Mathematics Beyond the Numbers* by George T. Gilbert and Rhonda L. Hatcher.

Notes on Section 4.3 *Greatest Common Divisors and Least Common Multiples*

- The Euclidean Algorithm can be pretty mystifying. I believe this lesson provides a good opportunity for your to discuss the teaching of algorithms. It is not uncommon for students in this course to confuse the ability to carry out an algorithm with understanding. I am not saying that we should be teaching them how to teach, only that this lesson can provide motivation for them to become teachers who develop algorithms from concepts. Anybody can apply the Euclidean Algorithm, but you much more likely to forget it if you do not understand it.

- Students may not understand why you would want to use the Euclidean Algorithm, if you only use the algorithm with small numbers. I like to have students find the GCD of two large numbers in several different ways (including the Euclidean Algorithm) so that they can see the advantage of the Euclidean Algorithm.

- I think it is good for students to do numbers 20–22 in Section 4.3 for homework. Many of my students had difficulty with word problems, but word problems really test their understanding of the concepts.