# Chapter 5
## Notes for Instructors

**Content**

The integers are the focus of this chapter. I only covered sections 5.1–5.3 of this chapter. Because I spent a lot of time on the properties of whole number arithmetic in chapter 2, the properties of integer were fairly evident to the students. I did point out the new properties that we obtain when we go from whole number arithmetic to integer arithmetic, but overall I spent very little time on the properties of integer arithmetic. (For example, I mentioned that the whole numbers are not closed under subtraction, but the integers are closed under subtraction.)

**Manipulatives**

Colored counters are the only manipulatives discussed extensively in Chapter 5. We do not have red and black colored counters, but we do have orange and yellow colored counters in the Mathskellar. I prefer the red an black counters because of their connection to the monetary phrases "in the red" and "in the black," but the students did not seem to mind the orange an yellow counters. We also have multicolored centimeter cubes in the Mathskellar. If you want to take a few minutes to separate the red and black cubes from the other colors, you could also use these for the colored counters.

**Notes and Suggestions**

**Notes on Section 5.1:** *Representations of Integers*

- It is important that students understand that there are many ways to represent an integer with colored counters. They will need to understand this if they are going to use take-away to demonstrate subtraction problems.

- Students often have difficulty with the definition for absolute value. They know that absolute values should be positive, but they fail to understand that $-a$ is positive when $a$ is negative.

**Notes on Section 5.2:** *Addition and Subtraction of Integers*

- The take-away model for subtraction of integers is considerably more difficult for students than the take-away model for subtraction of whole numbers. When we use colored counters and the take-away model to represent $a - b$, we will need to be certain that the colored counter representation we use for $a$ contains a colored counter representation for $b$. For example, if I want to evaluate $5 - (-3)$ using take-away and colored counters, I will need to have a colored counter representation for 5 which contains a colored counter representation for $-3$ so that I can take-away $-3$. So if I use five black colored counters to represent 5, then I cannot take away a representation for $-3$. On the other hand, if I use, for example, ten black counters and five red counters to represent 5, then I can take away three red counters which represents subtracting $-3$. This will leave me with ten black counters and two red counters. This group of colored counters represents 8. Hence $5 - (-3) = 8$.

- I think the difficulty students have with the take-away model for integer subtraction provides motivation for defining subtraction using the missing addend model. The take-away model which is easy to understand for whole number subtraction does not generalize easily to subtraction with other number systems.
- Many of my students had difficulty with the number line model that they use for subtraction. It is, in essence, illustrating the missing addend model for subtraction.

**Notes on Section 5.3:***Multiplication and Division of Integers*

- I do like the justification for integer multiplication in the textbook that uses patterns. I actually had my students work through a similar exercise in class.
- I really like to assign question 8 in section 5.3 for homework. I found that some students needed practice in identifying the properties of (integer) arithmetic.