Division Problems

PROBLEM: Is $8 \div 2$ defined or undefined? Use the missing factor model to explain your answer.

ANSWER: If $8 \div 2$ is defined then there is a unique whole number c such that

$$2 \times c = 8 \tag{1}$$

Since the only solution to equation (1) is $c = 4$, it follows that $8 \div 2 = 4$.

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PROBLEM: Is $8 \div 2$ defined or undefined? Use the repeated subtraction model to explain your answer.

ANSWER: Let us examine $8 \div 2$ by using the repeated subtraction model. Suppose that you have a pile of 8 coins. We need to remove groups of two coins from the pile of 8 coins until all the coins are gone. If $8 \div 2$ is defined, then it is defined to be the total number of groups we remove. Removing one group of two coins will leave us with a pile of 6 coins. Removing another group of two coins will leave us with a pile of 4 coins. Removing another group of two coins will leave us with a pile of 2 coins. Removing yet another group of two coins will leave us wil a pile of 0 coins. Since we had to remove 4 groups of two coins in order to remove all the coins from the original pile of 8 coins, it follows that $8 \div 2 = 4$.

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PROBLEM: Is $8 \div 2$ defined or undefined? Use the partition model to explain your answer.

ANSWER: Let's examine $8 \div 2$ according to the partition model. Suppose that we have 8 sandwiches and 2 lunch boxes. We need to place each sandwich in a lunch box, and we need to make sure that each lunch box contains the same number of sandwiches. If we can do this in a unique way, then $8 \div 2$ will be defined to be the number of sandwiches in each lunch box The only way to evenly distribute all of the sandwiches into the lunch boxes is to place 4 sandwiches in each lunch box. Therefore $8 \div 2 = 4$.

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PROBLEM: Is $56 \div 0$ defined or undefined? Use the missing factor model to explain your answer.

ANSWER: If $56 \div 0$ is defined, then there is a unique whole number c such that

$$0 \times c = 56. \tag{2}$$

Clearly equation (2) does not have any solutions. Therefore $56 \div 0$ must be undefined.

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PROBLEM: Is $56 \div 0$ defined or undefined? Use the repeated subtraction model to explain your answer.

ANSWER: Let's examine $56 \div 0$ according to the repeated subtraction model. Suppose we start with a group of 56 coins. We need to remove groups of zero coins from the pile of 56 coins until all the coins are gone. But removing groups of zero coins from the pile will always leave 56 coins in the pile. That is to say, we will never be able to remove all the coins from the pile in this way. Therefore $56 \div 0$ is undefined.

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PROBLEM: Is $56 \div 0$ defined or undefined? Use the partition model to explain your answer.

ANSWER: Let's examine $56 \div 0$ according to the partition model. Suppose we have 56 sandwiches and 0 lunch boxes. To evaluate $56 \div 0$ we would need to put each sandwich in a lunch box and make sure that each lunch box has the same number of sandwiches. Then we would need to count the number of sandwiches in each box. Since there are 0 bags, we cannot put each sandwich in a lunch bag. Therefore $56 \div 0$ is undefined.

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Note: $0 \div 0$ *is a difficult concept, and rightly so since it it undefined. I think that the missing factor model provides the simplest explanation as to why* $0 \div 0$ *is undefined. I have used all three models here, but I think the last two explanations are pretty convoluted.*

PROBLEM: Is $0 \div 0$ defined or undefined? Use the missing factor model to explain your answer.

ANSWER: If $0 \div 0$ is defined, then there is a *unique* whole number c such that

$$0 \times c = 0. \tag{3}$$

There are many solutions to equation 3. For example $c = 0$ and $c = 5$ are both solutions to equation 3. Therefore equation 3 does not have a *unique* solution and, hence, $0 \div 0$ is undefined.

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PROBLEM: Is $0 \div 0$ defined or undefined? Use the repeated subtraction model to explain your answer.

ANSWER: Let us examine $0 \div 0$ using the repeated subtraction model. Suppose that you have a pile of 0 coins. We need to ask ourselves how many groups of 0 coins must be removed from the pile so that no coins are left in the pile. Since the initial pile does not contain any coins, I could remove 0 groups of 0 coins and leave a pile with no coins. So at first glance it might appear that $0 \div 0$ is equal to zero, but let's not be hasty. I could also, for example, remove 3 groups of zero coins and leave a pile with no coins. From this example, it appears that $0 \div 0$ could also be equal to 3. Therefore, there is not a unique solution for $0 \div 0$, and, hence, $0 \div 0$ is undefined.

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PROBLEM: Is $0 \div 0$ defined or undefined? Use the partition model to explain your answer.

ANSWER: Let us examine $0 \div 0$ using the partition model. Suppose you have 0 sandwiches and 0 lunch boxes. We need to place each sandwich in a lunch box so that each lunch box contains the same number of sandwiches. Then we need to count the number of sandwiches in each lunch box. The total number of sandwiches in each lunch box would be our answer. However, we cannot count the number of sandwiches in each lunch box because we do not have any lunch boxes. Therefore $0 \div 0$ is undefined.