Sets Worksheet Name: ____________________________

MA 201

WARNING: You must **SHOW ALL OF YOUR WORK**. You will receive NO CREDIT if you do not show your work.

1. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ be the universe. Let $A = \{x \in U | \, x \text{ is even}\}$, $B = \{x \in U | \, 1 \leq x < 10\}$, and $C = \{x \in U | \, x \text{ is even or } x = 15\}$. Find the following.

   (a) $n(A)$

   (b) $n(B)$

   (c) $n(A \cup B)$

   (d) $n(A \cap B)$
   (What relationship do you notice between $n(A)$, $n(B)$, $n(A \cup B)$, and $n(A \cap B)$? Does this relationship always hold? If so, explain why it always holds. If not, provide a counterexample.)

   (e) $A \cap B$

   (f) $A \cup \overline{C}$

   (g) $\overline{A \cup C}$

   (h) $(A \cup \overline{B}) \cap C$

2. True or False.

   (a) ____________ $\{1, 2\} = \{2, 1\}$

   (b) ____________ $\{1, 2\} \sim \{2, 1\}$

   (c) ____________ $\{1, 2\} \sim \{3, 4\}$

   (d) ____________ $(1, 2) = (2, 1)$

   (e) ____________ Let $B = \{a, b, c, d, e\}$. Then $B = 5$.

   (f) ____________ $\emptyset \subseteq \{a, b, c\}$

   (g) ____________ $0 \div 21$ is defined.

   (h) ____________ $21 \div 0$ is defined.

   (i) ____________ $\{a, b, c\} \cup \emptyset = \{a, b, c\}$

   (j) ____________ $\{a, b, c\} \cap \emptyset = \{a, b, c\}$

   (k) ____________ $\emptyset = \{0\}$

   (l) ____________ $n(\emptyset) = 0$

   (m) ____________ Let $A = \{x | \, x \text{ is an even whole number}\}$.
   Let $B = \{y | \, y \text{ is an even natural number}\}$. Then $B \subseteq A$.

   (n) ____________ Let $A = \{x | \, x \text{ is an even whole number}\}$.
   Let $B = \{y | \, y \text{ is an even natural number}\}$. Then $B \subset A$.

3. True or False. If the statement is true, briefly explain why it is true. If it is false, provide a counterexample.

   (a) ____________ If A and B are finite sets, then $n(A) + n(B) = n(A \cup B)$.

   (b) ____________ If A and B are finite sets, then $n(A) \times n(B) = n(A \times B)$

   (c) ____________ If $n(A \cap B) < n(A)$, then $B \subset A$.

   (d) ____________ If $A \subseteq B$ and $B \subset C$, then $A \subset C$.

4. (a) Show that the set of whole numbers, W, is equivalent to the set of natural numbers, N, by carefully describing a one-to-one correspondence between the sets.

   (b) According to the one-to-one correspondence you described in part (a), which whole number is paired with the natural number 999?

   (c) According to the one-to-one correspondence you described in part (a), which natural number is paired with the whole number 999?

   (d) According to the one-to-one correspondence you described in part (a), which whole number is paired with the natural number x?

   (e) According to the one-to-one correspondence you described in part (a), which natural number is paired with the whole number y?

5. (a) Show that the set of even whole numbers, E, is equivalent to the set of odd whole numbers, O, by carefully describing a one-to-one correspondence between the sets.

   (b) According to the one-to-one correspondence you described in part (a), which even number is paired with the odd number 999?

   (c) According to the one-to-one correspondence you described in part (a), which odd number is paired with the even number 764?

   (d) According to the one-to-one correspondence you described in part (a), which even number is paired with the odd number m?

   (e) According to the one-to-one correspondence you described in part (a), which odd number is paired with the even number n?

6. Do number 11 on page 94 of your textbook.

7. Let $A = \{a, b, c\}$ and $B = \{c, d, ef\}$. Then $n(A) = 3$, $n(B) = 4$, and $n(A \cup B) = 6$ (Why?) Look at the definition for addition of whole numbers given on page 99 of your textbook. In this example, $n(A) + n(B) \neq n(A \cup B)$. Is there a conflict between this example and the definition for addition of whole numbers? Why or why not? Explain briefly.

8. Use the measurement model to illustrate the following.

   (a) $4 + 6 = 6 + 4$

   (b) $4 \times (2 + 3) = 4 \times 2 + 4 \times 3$

9. True or False. Briefly justify your answer.

(a) ____________ $\{0, 1\}$ is closed under multiplication.

(b) ____________ $\{0, 1\}$ is closed under subtraction.

(c) ____________ $\{0, 1\}$ is closed under addition.

(d) ____________ Let X be s subset of the whole numbers that contains 2 and 4. If X is closed under addition then 3 cannot be an element of X.

10. Let X be a subset of the whole numbers that contains 2. If X is closed under addition, what whole numbers must be contained in X? What, if any, numbers are certainly not contained in X?

11. Use the number line to illustrate the following facts.

    (a) $15 - 2 = 13$

    (b) $4 * 5 = 20$

12. Do number 16 on page 109 of your textbook.

13. Do number 28 on page 111 of your textbook.

14. State the Division Algorithm.

15. Use sets to show that $6 < 9$.

16. Do units need to be the same when adding? when subtracting? when multiplying? when dividing?

17. For each subtraction model, write a separate word problem that illustrates

$$53 - 26.$$

    (a) The comparison model for subtraction

    (b) The missing addend model for subtraction

    (c) The take-away model for subtraction

    (d) The measurement model for subtraction

18. For each addition model, write a separate word problem that illustrates

$$53 + 26.$$

    (a) The set model for addition

    (b) The measurement model for addition

19. For each multiplication model, write a separate word problem that illustrates

$$4 \times 3.$$

    (a) The repeated addition model for multiplication

(b) The array model for multiplication

(c) The rectangular area model for multiplication

(d) The multiplication tree model for multiplication

(e) The Cartesian product model for multiplication

20. For each division model, write a separate word problem that illustrates

$$35 \div 7.$$

(a) The repeated subtraction model for division

(b) The partition model for division

21. Clearly explain why division by zero is undefined.

22. Clearly explain why $\frac{0}{253}$ is defined.

23. Rewrite as a single whole number exponential, if possible.

(a) $\dfrac{7^{15}}{7^4}$

(b) $2^5 - 2^3$

(c) $(2^5)^6$

(d) $4^5 \times 6^5$

24. Evaluate 3^0.

25. Do number 25 on page 131 of your textbook.

26. Let $X = \{1, 2, 3\}$ and $Y = \{a, b\}$.

(a) Find $X \times Y$.

(b) Find $Y \times X$.

(c) Is $X \times Y = Y \times X$?

(d) Is $n(X \times Y) = n(Y \times X)$?

27. Let $X = \{a, b\}$ and $Y = \{b, a\}$.

(a) Find $X \times Y$.

(b) Find $Y \times X$.

(c) Is $X \times Y = Y \times X$?

(d) Is $n(X \times Y) = n(Y \times X)$?

28. Let A and B be finite sets.

(a) What can be said about $n(A \times B)$ and $n(B \times A)$? Relate this observation to a property of whole number multiplication.

(b) If $A \times B = B \times A$, what must be true about A and B?