Chapter 8
Notes for Instructors

Content

Algebra is the focus of this chapter. Students should be able to:

1. recognize patterns
2. write an algebraic expression for the n^{th} term in a sequence (which may be given as a sequence of geometric diagrams)
3. write an algebraic expressions for a situation posed in a word problem
4. use the equality sign "=" appropriately
5. solve simple equations
6. understand the concept of a function as a formula, as a table, as a machine, and as a graph
7. understand the concepts of domain and range
8. explain the vertical line test
9. identify points in the Cartesian Coordinate System
10. apply the Pythagorean Theorem to find the distance between two points in the Cartesian Coordinate System. (*I am trying to help my students understand the Distance Formula. Most people remember the Pythagorean Theorem. Few rember the Distance Formula. I want my students to be able to solve distance problems even if they forget the Distance Formula.*)
11. understand that the slope of a line is invariant. (*This could be used as the definition of a line.*)
12. write the equation of a given line
13. graph linear equations
14. understand the Condition for Parallelism

I spent about two weeks on this chapter, but I certainly could have spent much more time. I really think that I should not have spent more than one day reviewing MA 201 material at the beginning of the course. I think it would have been better to review throughout the semester. It was hard to teach this group of students because many had not had MA 201 in its present form. Even so, I think it would have been best to jump right into the new material and review as it became necessary.

Manipulatives

There are manipulatives that can be used for Algebra. I do not believe that we currently have any of these manipulatives at UK. Nevertheless, there are some ideas which can be used to make concepts in these chapters easier for students. I found that the traditional notion of a function as a machine was quite useful (as you will see in the worksheet for this chapter).

Notes and Suggestions:

Notes on Section 8.1: *Algebraic Expressions and Equations*

- A lot of the students in this course have difficulty with definitions. Consequently, they have difficulty writing what they mean. For example, when my class was studying the Geometry chapter, I saw the following sentences: "The sum of the angles is 360°" and, worse yet, "The angles are 360°." My students meant to (but did not) say that the sum of the *measures* of the angles is 360°." On another section in the Geometry chapter, my students had a lot of difficulty with a question regarding regular polygons. They did not read the section carefully. Consequently, they did not know that the regular polygons are a proper subset of the polygons. They just don't fully understand the precision involved with definitions. I tell you this a few chapters early because I believe you need to stress the importance of of precise and acurate writing from the very first day of this class. In particular, with respect to section 8.1, students should be very careful with the "=" sign. It should not be used to represent "implies that." They should only use it when quantities are indeed equal. For example, suppose a student must square 3 and then add 1. It is not uncommon to see the following work on a student's paper: "$3^2 = 9 + 1 = 10$. Clearly, this is false and should be corrected. I think your only hope is to start early and to deduct points. I am not saying that you need to deduct a lot. Usually, a deduction of half a point will get their attention without really affecting their grade.

- I really like the rate problem in Example 8.5 on pages 484–485, but there are not any rate problems in the exercises. I have included several rate problems on the Traditionally, students have difficulty with these types of problems. I like these problems because they involve fractions which is a good review. I also like these problems because they provide a good writing exercise. Students should not be given credit for simply solving an equation. They should explain how they arrived at the equation that they must solve.

- I am fond of problem 13 in Section 8.1.

Notes on Section 8.2: *Funtions*

- I found that it was very useful to view functions as machines. I like to use the machines to write compound functions. You can then work your way backwards through the machines to solve simple equations. For example, if we want to solve $2(x+1)^2 = 8$, we write the lefthand side as a sequence of machines. First x goes into the $+1$-machine.

From there it goes into the squaring machine. Finally, it goex into the $\times 2$-machine. So, if we need to solve this equation, we must first divide both sides of the equation by 2 (which undoes the $\times 2$-machine), then we take the square root of both sides of the equation (which undoes the squaring machine), and finally we subtract 1 from both sides of the equation (which undoes the $+1$-machine).

- Students know about the vertical line test, but they should be able to explain the vertical line test.

- I like Problems 3, 9–11, and 15–16 in Section 8.2.

Notes on Section 8.3: *Graphing Functions in the Cartesian Plane*

- I think it is a good idea to pass out the graphs of several lines to the class. Make sure to include a vertical line, a horizontal line, and a pair of parallel lines. Choose several points on one of the lines. Calculate the slope of the line using different pairs of points. Students should come to the conclusion that the slope is invariant. Have them find the slopes of the parallel lines. Perhaps you could also include a pair of perpendicular lines. Have them find several points on the horizontal line and the vertical line. What is true of the y values on the horizontal line? What is true of the x values on the vertical line? I have included a graph containing several lines with this documentation.

- I found that my students had been graphing equations for years without truly understand what the graph of an graph is. It might not hurt to mention that a point is on the graph if and only if it satisfies the equation.

- I think it is really important that students in this class stop memorizing formulas with the exception of some basic formulas. Students know the Pythagorean Theorem, but they forget the Distance Formula (or they cram it before a test). I think it is important that they be able to use the Pythagorean Theorem to find the distance between two points. I had my students plot two specific points and draw an appropriate right triangle. They had to do this on the test.

Worksheets

I have included one worksheet with this documentation.