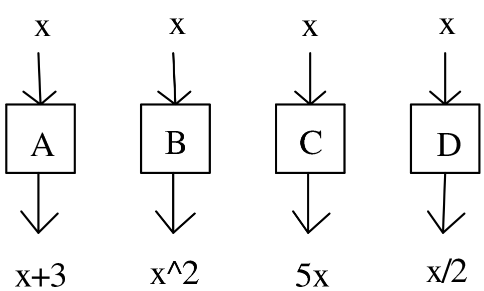
FUNCTIONS WORKSHEET Name:________________________

MA 202
Spring Semester 2004

WARNING: You must **SHOW ALL OF YOUR WORK**. You will receive NO CREDIT if you do not show your work.
DUE: Thursday, 05 February 2003

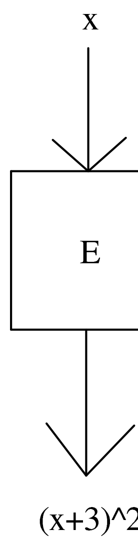
1. Jake is is y years old. Write an algebraic expression with the variable y for Joni's age if:
   (a) Joni is three years older than Jake was five years ago.
   (b) Joni is twice as old as Jake will be in six years.
   (c) Joni is half as old as Jake's mother who is three times as old as Jake was four years ago.

2. Consider the four number machines shown below.

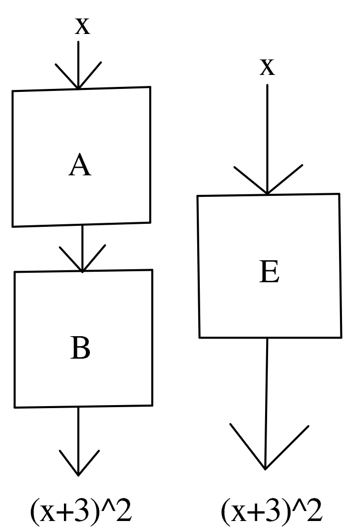


Machine A takes a number x and returns $x + 3$. For example, if we place the number 5 in machine A, it will return the number 8. If we place the number -1.2 in machine A, it will return the number 1.8. Machines B, C, and D act similarly. If we place the number 4 in machine B, it will return the number ____________. If we place the number -7 in machine C, it will return the number ____________. If we place the number 5.2 in machine D, it will return the number ____________.

Machines A, B, C, and D are *simple machines* because they only perform one operation on a number. Let's take a look at a machine which is not a simple machine. Consider machine E which is shown below.



Machine E is a *complex machine* because it performs more than one operation on the number that is input. If we place the number x into machine E, it will first add three to x and then square $(x+3)$ yielding $(x+3)^2$. We can construct E from simple machines. For example, we can input a number into machine A and then place the output from machine A into machine B. When machines A and B are linked in this way, together they perform the same operations that machine E performs.



I have described several complex machines below. You need to construct each complex machine from the simple machines A, B, C, and D. Your answer should be a diagram similar to the ones shown above.

- When the number x is placed in machine F, it returns $\dfrac{(x+3)^2}{2}$.

- When the number x is placed in machine G, it returns $\dfrac{x^2+3}{2}$.
- When the number x is placed in machine H, it returns $5x+3$.
- When the number x is placed in machine I, it returns $5(x+3)$.
- When the number x is placed in machine J, it returns $\dfrac{(5x+3)^2}{2}$.

Mathematicians are, by nature, somewhat lazy when it comes to writing. We really like shorthand — provided that it is used correctly. (Remember that you need to be very, very careful with the $=$ symbol. You may only use it to mean that quantities are, in fact, equal. You may not use this symbol for the word "implies.") It requires a lot of time to draw number machines, especially if you are using a computer as I have been doing to write this worksheet. The shorthand for number machines is called *function notation*. Here is how it works. Consider number machine A. The name of this machine is "A." It takes the input x and returns the value $x+3$. Therefore the shorthand for machine A is $A(x) = x+3$. Since $A(x) = x+3$, we can either say that the output of machine A is $x+3$ or $A(x)$. If we want to put a specific number into the machine we can replace the input variable x with our number. For example, if we put the number 5 into machine A, it will return the value 8. We represent this by $A(5) = 8$. Notice that $A(5) = 5+3$, that is 5 replaces *all* of the x values in our notation.

- Write the function notation for machines B–J.
- What is $J(2)$?
- What is $C(-5)$?
- What is $D(4)$?
- What is $I(x^2+h)$?
- What is $B(x+3)$?
- What is $B(A(x))$?

Recall that we can construct machines E–J from our simple machines. How would we represent this using function notation? Let's take a look at machine E. We constructed E from simple machines by joining machine A to machine B. We input the value x into machine A and it returns the value $A(x)$. Then this value goes into machine B, that is $A(x)$ goes into machine B. Therefore B should return the value $B(A(x))$. Hence, we can write $E(x) = B(A(x))$. This function notation is used to show how the complex machine E can be constructed from the simple machines A and B.

- Each of the complex machines F–J can be constructed from the simple machines A–D. Describe these constructions using function notation.

Now let's see how we can use these machines to help us solve equations. Suppose we want to solve

$$(x+3)^2 = 5.$$

Notice that all the x's are on one side of the equation. Also notice that the left hand side of the equation is $E(x)$. To solve this equation, we need to walk backwards through the simple machine construction. Remember, that we constructed E from simple machines by sending a number through machine A and then through B. So if we are going to walk backwards through this construction we will need to undo the operation in B and then undo the operation in A. Since B squares a number, we will need to undo this operation by taking the square root of both sides of the equation. Note that we will have to be careful here to include both the positive and negative square root. This gives:

$$\begin{aligned} (x+3)^2 &= 5 \\ \sqrt{(x+3)^2} &= \sqrt{5} \\ x+3 &= \pm\sqrt{5} \end{aligned}$$

Now we need to undo the operation of machine A. Since A adds three to a number, we will need to undo this operation by subtracting three from both sides of the equation. This gives:

$$\begin{aligned} (x+3)^2 &= 5 \\ \sqrt{(x+3)^2} &= \sqrt{5} \\ x+3 &= \pm\sqrt{5} \\ x+3-3 &= \pm\sqrt{5}-3 \\ x &= \pm\sqrt{5}-3 \end{aligned}$$

- Verify that $\sqrt{5}-3$ and $-\sqrt{5}-3$ are both solutions to the equation $(x+3)^2 = 5$.

We can use this process of walking backwards through simple machines to solve lots of equations. Solve the following equations. Explain to your group members how your solutions are related to walking backwards through simple machines.

- $16 = \dfrac{(x+3)^2}{2}$.
- $\dfrac{1}{2} = \dfrac{x^2+3}{2}$.
- $x + 4.2 = 6x + 3$.
- $5(x+3) = 7$.
- $\dfrac{(5x+3)^2}{2} = \frac{1}{3}$.

3. For each complex function described below, describe the simple machines that are needed to construct the complex function. Then construct the complex machine from the simple machines you described.

(a) $k(x) = \frac{2x+3}{5}$

(b) $l(x) = \frac{1}{x+2}$

(c) $m(x) = \sqrt{x^2 - 1}$

(d) $n(x) = 3\sqrt{4x-5} - 7$

4. Find the solution set for each equation. Be sure to show all of your work and check your answers.

(a) $4x + 6 = 9x - 2$

(b) $\frac{2x+3}{5} = \frac{1}{10}$

(c) $\frac{1}{x+2} = 7$

(d) $\frac{1}{x+2} = 0$

(e) $\sqrt{x^2 - 1} = 0$

(f) $3\sqrt{4x-5} - 7 = -1$

5. Which of the following equations are identities? Which are conditional equations? Which equations do not have real number solutions?

(a) $x + 2 = 53$

(b) $2x + x^2 = x(2 + x)$

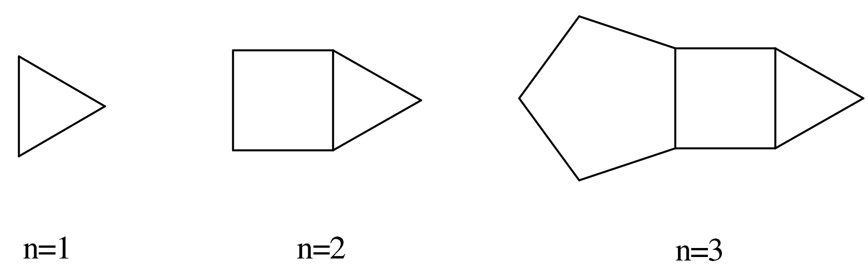
(c) $(x + y)^2 = x^2 + y^2$ (*Be careful here.*)

(d) $x^2 = \dfrac{1}{\frac{1}{x^2}}$

(e) $x^2 + 1 = 0.$

(f) $\sqrt{x^2 - 1} = 0$

6. Consider the sequence of polygon trains shown below. Write an algebraic expression using the variable n to represent the number of edges in the n^{th} polygon train.



7. Joni can mow Grandma's yard in 4 hours. Jake can mow Grandma's yard in 6 hours. How long will it take Jake and Joni to mow Grandma's yard if they work together?

8. Amy can make seven necklaces in one hour. Katie can make thirteen necklaces in two hours. How long will it take Amy and Katie to make fifty necklaces if they work together?

9. Consider the function $y = f(x)$.

   (a) **Circle the correct responses.** Suppose we want to graph this function in the Cartesian plane. The x values represent the (input, output) of the function. The y values represent the (input, output) of the function.

   (b) Describe the vertical line test. Explain why the vertical line test works when we consider functions of the form $y = f(x)$.

   (c) How would you have to adjust the vertical line test if we wished to consider functions of the form $x = f(y)$.

10. (a) Write a function $A(r)$ for the area of a circle of radius r.

   (b) Write a function $B(d)$ for the area of a circle of radius d.

   (c) Find $A(2)$, $A(4)$, $A(3)$, $A(6)$, $A(.5)$, and $A(1)$. What do each of these values represent?

   (d) How does the area of a circle change when we double its radius? Verify your answer using function notation. (*Hint:* If we double the radius r, then the new radius is expressed by ____________. Input the new radius into your area function A.)

   (e) How does the area of a circle change when we double its diameter. Verify your answer using function notation.

   (f) Which is the better deal: three ten inch pizzas for \$20 or one twenty inch pizza for \$20?

   (g) Write a function $P(r)$ for the perimeter of a circle with radius r.

   (h) How does the perimeter of a circle change when we double its radius? Verify your answer using function notation.

11. Use the Pythagorean Theorem to find the distance between $(4, 5)$ and $(7, 16)$.

12. What do points on a horizontal line have in common?

13. What do points on a vertical line have in common?

14. Write an equation for the horizontal line through the point $(12, 9)$.

15. Write an equation for the vertical line through the point $(12, 9)$.

16. Consider the graph of the parabola given by $f(x) = x^2$.

   (a) Find the slope of the line through the points of the parabola at $x = 2$ and $x = 3$.

(b) Find the slope of the line through the points of the parabola at $x = -2$ and $x = -3$.

(c) How would your results for this problem be different if $f(x)$ had described a line?

17. Write an equation for a line which passes through the point $(4, -1)$ and has slope 6.

18. Write an equation for the line which passes through the points $(-1, -2)$ and $(5, 6)$.

19. Write an equation for the line which passes through the points $(-1, -2)$ and $(-1, 6)$.

20. Write an equation for the line which passes through the points $(-1, 6)$ and $(5, 6)$.

21. Write an equation for the line which passes through $(-4, -5)$ and is parallel to the line $4x = 6 + 9y$.

22. Read Sections 8.1–8.3 of your text.