Class Notes

Contents

1 Thursday, August 27

1. Went over the syllabus.

2. Worked on Investigation 1 of Covering and Surrounding.

3. Examined the concepts of area and perimeter by constructing some polyominos.

4. Looked at the formulas for the area and perimeter of rectangles, and why these formulas make sense. If a rectangle has side lengths a and b, then its area is $A = ab$ and its perimeter is $P = 2a + 2b$.

5. Considered the problem of rectangles with minimum and maximum perimeter for a given area. For example, consider the area $A = 24\,\text{m}^2$. With whole number sides, the rectangle with the minimum perimeter has dimensions $4\,\text{m} \times 6\,\text{m}$, and the rectangle with the maximum perimeter has dimensions $1\,\text{m} \times 24\,\text{m}$. If we allow noninteger side lengths, then we can use algebra to write the perimeter as a function of one side length x:

$$xy = 24,$$
$$y = 24/x,$$
$$P = 2x + 2y = 2x + 2(24/x) = 2x + 48/x.$$

   We graphed this with Desmos, and saw that the minimum perimeter rectangle is a square with side length $\sqrt{24}\,\text{m}$, and that we can find rectangles with arbitrarily large perimeters.

6. Considered the problem of rectangles with minimum and maximum area for a given perimeter. For example, consider the perimeter $P = 24\,\text{m}$. With whole number sides, the rectangle with the maximum area has dimensions $6\,\text{m} \times 6\,\text{m}$, and the rectangle with the minimum area has dimensions $1\,\text{m} \times 11\,\text{m}$. If we allow noninteger side lengths, then we can use algebra to write the area as a function of one side length x:

$$2x + 2y = 24,$$
$$y = 12 - x,$$
$$A = xy = x(12 - x).$$

   We graphed this with Desmos, recognized that it is a parabola, and saw that the maximum rectangle is a square with side length $6\,\text{m}$, and that we can find rectangles with arbitrarily small positive areas. If we choose $x = 0\,\text{m}$ we obtain the degenerate rectangle with area zero.

7. Considered the problem estimating areas of arbitrary shapes by overlaying grids.

8. Visually confirmed that a one inch square consists of 4 half inch squares and 16 quarter inch squares. We expect this, since

$$\frac{1}{2}\,\text{in} \times \frac{1}{2}\,\text{in} = \frac{1}{4}\,\text{in}^2,$$

   and

$$\frac{1}{4}\,\text{in} \times \frac{1}{4}\,\text{in} = \frac{1}{16}\,\text{in}^2.$$

9. Showed how to convert units by "multiplying by 1s". For example, to convert 60 miles per hour to feet per second,

$$\frac{60\,\text{mi}}{1\,\text{hr}} \times \frac{5280\,\text{ft}}{1\,\text{mi}} \times \frac{1\,\text{hr}}{60\,\text{min}} \times \frac{1\,\text{min}}{60\,\text{sec}} = \frac{60 \cdot 5280\,\text{ft}}{60 \cdot 60\,\text{sec}}.$$

   As an example of converting square units, to convert 10 square yards to square feet,

$$10\,\text{yd}^2 \times \left(\frac{3\,\text{ft}}{1\,\text{yd}}\right)^2 = 90\,\text{ft}^2.$$

2 Tuesday, September 1

1. Discussed how to approach writing out solutions to problems. In particular, use full sentences and write in such a way that it is understandable to a friend or a student when read out loud. Include justifications for your statements, even if they are brief.

2. Worked on the Circle Area Problem on the course website. Saw, by drawing in some diagonals of the four smaller squares, that the area of a circle is larger than $2r^2$ but smaller than $4r^2$. In fact it is close to $3r^2$, as we know from the formula for the area of a circle, $A = \pi r^2$. If we cut up the "r-squares" to cover the circle, we will see that we will need about three of them.

3. Worked on the Rectangle Area Problem on the course website. Here we pretend we do not know the formula for the area of the rectangle yet, but by making a puzzle, adding up areas of two squares and two copies of the rectangle, and knowing the area formula for squares, we find that

$$a^2 + b^2 + 2R = (a+b)^2$$
$$a^2 + b^2 + 2R = a^2 + 2ab + b^2$$
$$2R = 2ab$$
$$R = ab$$

   So in this way we can justify the formula for the area of a rectangle. Note that we are making the assumption that the area of a rectangle does not change if we "move it around."

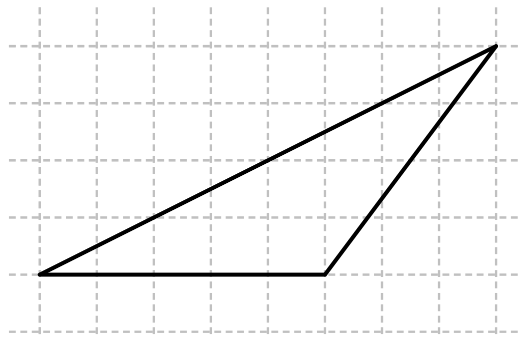
4. Demonstrated the website for calculating area and perimeter of a region within Google maps—see link on course website.

5. Pointed out the two iPad apps on the website for calculating area and perimeter of regions on maps.

6. Pointed out the two lessons involving area and perimeter that are listed on the website.

7. Worked on Investigation 2.1. You should review the terms defined at the beginning of this investigation. Saw that for these particular triangles, we can enclose them in rectangles and use this to verify that in each case the area of the triangle is half of the area of the rectangle, and so equals $\frac{1}{2}$base$\times$height.

8. Began working on figuring out the area of the following triangle, by enclosing it in a rectangle:

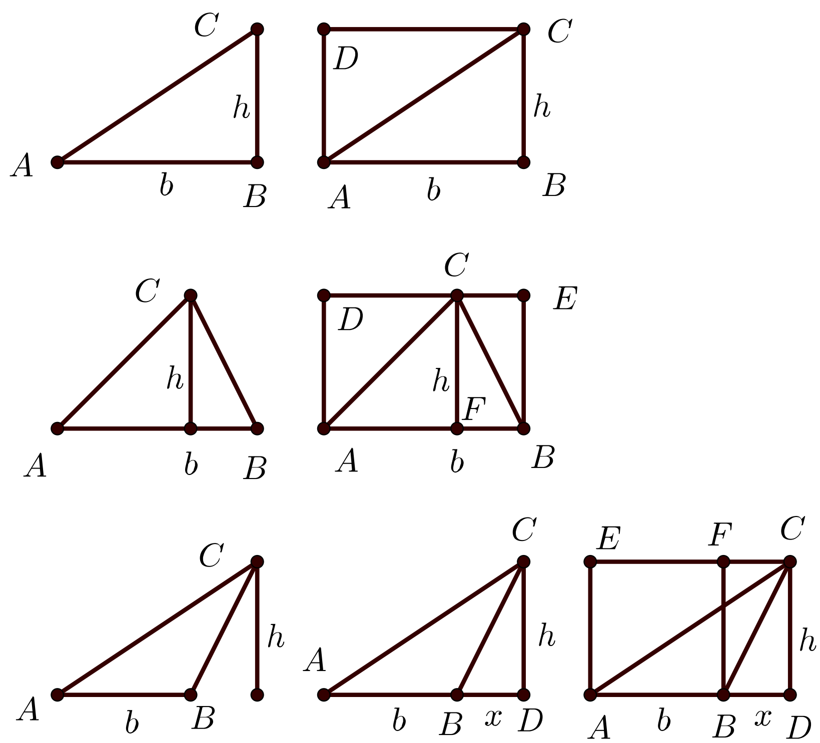


I asked everyone to keep working on this so that we can discuss solutions on Thursday.

3 Thursday, September 3

1. Noted that any side of a triangle can be used as a base, and that once a base is chosen, the height is a perpendicular line segment from the opposite vertex to the line containing the base.

   Finished up understanding how the formula for the area of a triangle can be deduced from the area of a rectangle. We considered three cases. See the figure below.

   In the first case, the triangle is a right triangle and the height ends up being one of the sides of the triangle. In this case the area of the triangle is seen to be half of the area of the enclosing rectangle $ABCD$ which is bh. So the area of the triangle is $\frac{1}{2}bh$.

   In the second case, the height of the triangle falls within the triangle. In this case the height breaks up the triangle and the enclosing rectangle each into two pieces. In each side the area of the piece of the triangle is half of the area of the piece of the rectangle, so overall the area of the entire triangle is half the area of the entire rectangle. Once again, the area of the triangle is thus $\frac{1}{2}bh$.

In the third case, the height of the triangle falls outside of the triangle. One approach is to see that the area of ΔABC equals the area of ΔADC minus the area of ΔBDC. These latter triangle are right, so we already know the formula for their areas. So the area of triangle ΔABC equals

$$\frac{1}{2}(b+x)h - \frac{1}{2}xh = \frac{1}{2}bh + \frac{1}{2}xh - \frac{1}{2}xh = \frac{1}{2}bh.$$

Another approach is to see that the area of ΔABC is half of the area of rectangle $ADCE$ minus half of the rectangle $ABFE$. This results in the same formula.

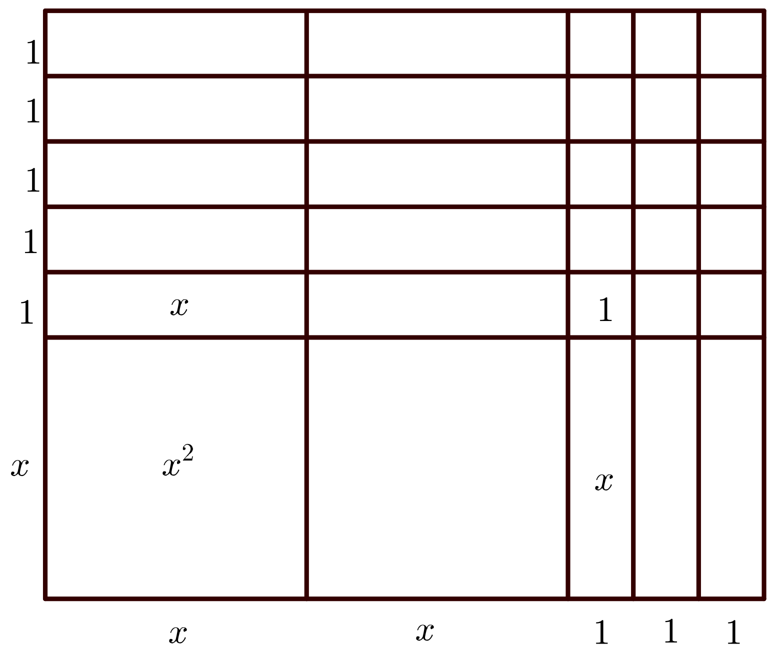
So in all three cases we have demonstrated that the area of a triangle with base b and height h is $\frac{1}{2}bh$.

2. Considered parallelograms. We assume we know some familiar properties, such as they they are quadrilaterals, and each pair of opposite sides are parallel and congruent. Again, any side may be regarded as a base, and the corresponding height can be determined by drawing a perpendicular line segment from one of the other two vertices to the line containing the base. We saw that if we have a parallelogram with base b and height h, we can divide it by a diagonal into two triangles, each with base b and height h. Each triangle has area $\frac{1}{2}bh$, so the area of the paralellogram is bh.

   Another approach (see Probem 3.1D) is to cut off a triangle from one end of the parallelogram and glue it onto the other end. But this process might not work if the parallelogram "leans" to far, such as parallelogram F in Problem 3.1. As we saw in class, this kind of parallelogram could first be sliced up by horizontal llines into pieces each small enough for this "triangle transfer" process to work.

3. Note that we have now worked through a nice progression: starting with the formula for the area of a square we derived and justified the formula for the area of a rectangle, and then a triangle, and then a parallelogram.

4. Mentioned that the area model for multiplication is a very powerful visusalization tool. On a simple level, you can visualize, e.g., that $3\times 7 = 21$. But you can also visualize the multiplication of certain algebraic expressions, such as $(x+5)(2x+3) = 2x^2+13x+15$:



5. Talked about the importance of fostering a "growth mindset" among your students, rather than a "fixed mindset." Here is an article about this: `https://alumni.stanford.edu/get/page/magazine/article/?article_id=32124`. And here is a video about the London cabbies: `https://www.youtube.com/watch?v=i9JPkUE2IJw`.

6. Demonstrated GeoGebra, a very useful tool for visualizing concepts and constructions in geometry and algebra. You can download it or run it the web. See `https://www.geogebra.org`.

7. Looked at the *Common Core State Standards for Mathematics*, `http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf`. In particular, we looked at the sixth grade geometry standards, and saw that the material in *Covering and Surrounding* addresses these standards. A class member mentioned a very good course site to see illustrations of the standards, Illustrative Mathematics, `https://www.illustrativemathematics.org`.

4 Tuesday, September 8

1. Looked at the notion of prisms, both right and oblique. Note that if the pair of parallel congrurent bases of a prism are not themselves parallelograms, then there is no other choice of bases, but if the prism is a right or oblique prism over a parallelogram, then any pair of opposite faces can be viewed as bases.

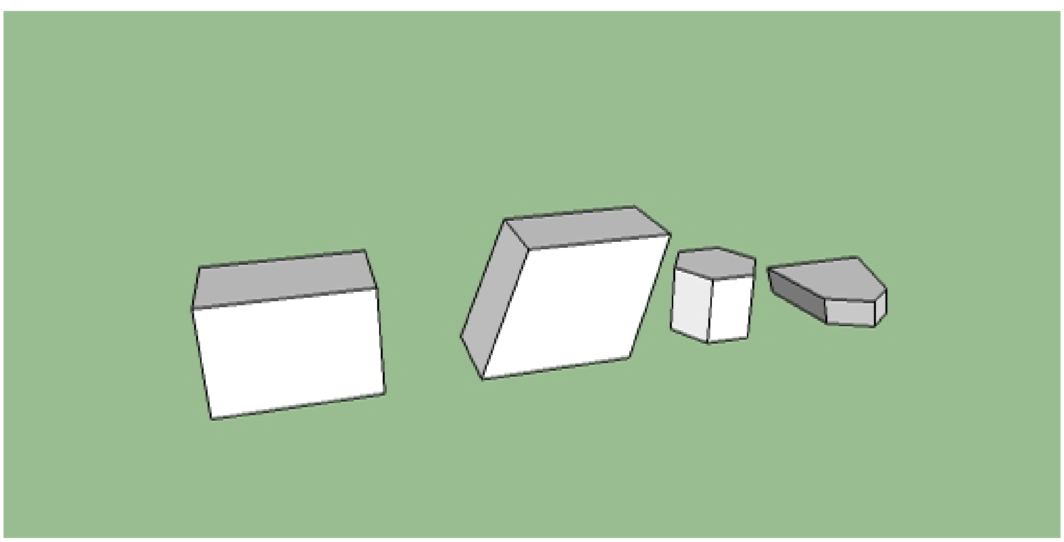
   We may refer to the area of the base as B.

   The height H of a prism is determined by the length of a line segment joining and perpendicular to the two planes containing the two bases.

   A special type of prism is a right rectangular prism, which informally is a "box".

   We constructed prisms with the manipulative Polydron.

2. Demonstrated the program SketchUp, which provides a very convenient way to sketch three-dimensional shapes in general, and prisms in particular.



3. In general, we justified by stacking up cubes that the volume of a general prism is the area of its base multiplied by its height, BH.

   For a right rectangular prism with dimensions a, b, c in particular, the above formula becomes $V = abc$, which is again seen to make sense by making and folding up nets and filling them with cm cubes in the case that a, b, c are whole numbers.

   Note that volume is measured in cubic units.

4. In general, we saw that the surface area of a general prism can be obtained by adding up the areas of the two bases and all of the lateral faces. This can be approximated by unfolding and making a net out of the (hollow) prism and determining the areas of all of the resulting polygons.

   For a right rectangular prism with dimensions a, b, c in particular, we justified the formula $S = 2ab + 2ac + 2bc$.

   Note that surface area is measured in square units.

5. Note that you must be careful not to confuse the height H and base area B of a prism with the height h and base b of one of the two-dimensional faces or bases of the prism.

5 Thursday, September 10

1. Worked on #1 and #4 of the "Polygon Area Problems" handout. NOTE: I should have specified that we are working with RIGHT prisms. We found the area of the equilateral triangle by drawing a height of length h, which bisected the triangle and created a right triangle with sides $s/2$ and h, and hypotenuse s. Thus, using the Pythagorean Theorem,

$$\left(\frac{1}{2}s\right)^2 + h^2 = s^2$$

$$\frac{1}{4}s^2 + h^2 = s^2$$

$$h^2 = s^2 - \frac{1}{4}s^2 = s^2\left(1 - \frac{1}{4}\right) = s^2\frac{3}{4}$$

$$h = \sqrt{s^2\frac{3}{4}} = \frac{s}{2}\sqrt{3}.$$

   Thus the area of the triangle is

$$A = \frac{1}{2}sh = \frac{s^2}{2}\sqrt{3}.$$

   Now the volume of the prism is the area of the base of the prism multiplied by the height of the prism, so

$$V = BH = \frac{s^2}{2}\sqrt{3}H.$$

   The surface area of the prism comes from the two bases (equilateral triangles) and the three lateral $s \times H$ rectangles, so

$$S = 2\frac{s^2}{2}\sqrt{3} + 3sH = s^2\sqrt{3} + 3sH.$$

2. Did an exercise in class to help everyone learn everyone's name.

3. Worked on classifying triangles: acute, right, obtuse, scalene, isosceles, equilateral. See the diagram that I emailed out after class.

6 Tuesday, September 15

1. Looked at diagrams to classify triangles and quadrilaterals.

2. Measured and tore off the angles of triangles to gather evidence that the interior angle sum appeared to be 180 degrees.

3. Assuming that the interior angle sum of every triangle is 180 degrees, proved that the interior angle sum of any quadrilateral, convex or nonconvex, was 360 degrees, by dividing the interior into two triangles.

4. Proved that the interior angle sum of every convex n-gon is $180(n-2)$. We had two proofs. For the first proof we chose one vertex and drew $n-3$ diagonals from this vertex to each of its non-neighbors. This divided the polygon into $n-2$ triangles. The sum of the interior angles of all of these triangles was seen to be the sum of the interior angles of the polygon. For the second proof we inserted a new point in the interior of the n-gon and joined this point to each of the n vertices, creating n triangles. We saw that the sum of the interior angles of all of these triangles was equal to 360 degrees plus the sum of the interior angles of the polygon. So this latter sum equals $n180 - 360 = (n-2)180$ degrees.

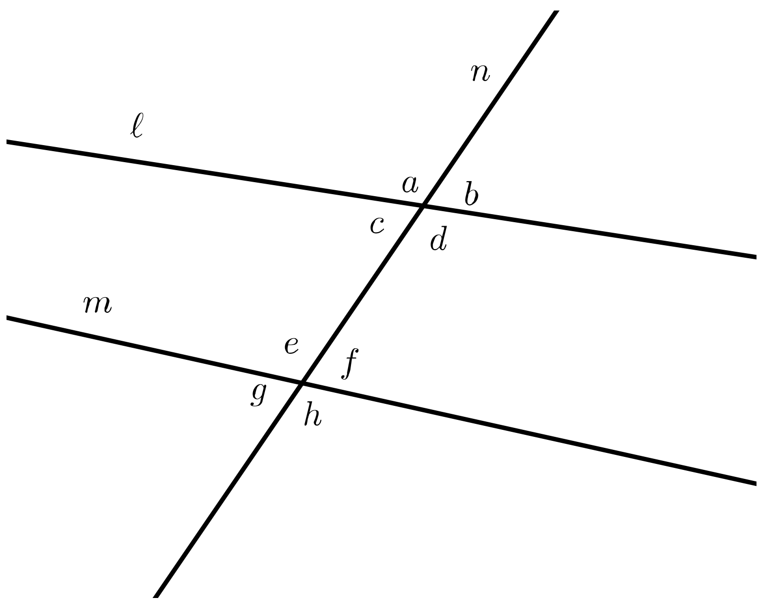
   Note: It turns out, though we did not prove it, that even if an n-gon is not convex, the interior can be divided up into $n-2$ triangles, so the interior angle sum of any n-gon is $180(n-2)$ degrees.

5. We proved that the exterior angle sum of every polygon is 360 degrees. For the first proof, we observed how to move a line segment (in our case, my pen) around the exterior to observed that it rotated exactly once. For the second proof, we used the fact that at each vertex the exterior angle is supplementary to the interior angle, so we can use algebra to determine the exterior angle sum from the interior angle sum.

7 Thursday, September 17

1. Recalled that when two lines cross, each pair of opposite angles forms a vertical pair of angles with equal measure, and each pair of adjacent angles is supplementary.

2. Consider the figure below.



   Line n is a transversal to lines ℓ and m. Angles a and d form a pair of vertical angles, so $a = d$. Angles a and c form a pair of supplementary angles, so $a + c = 180$ degrees.

   Angles a and e form a pair of corresponding angles. Angles a and h form a pair of alternate exterior angles. Angles c and f form a pair of alternate interior angles. Angles c and e form a pair of consecutive interior angles.

3. We will assume the following theorems to be true:

   Theorem 1. If x and y are a pair of corresponding angles in a transversal n of ℓ and m, and if ℓ and m are parallel, then $x = y$.

   Theorem 2. If x and y are a pair of corresponding angles in a transversal n of ℓ and m, and if $x = y$, then ℓ and m are parallel.

We can use these two theorems, together with vertical angles and supplementary angles, to prove:

- If x and y are a pair of alternate exterior angles in a transversal of parallel lines, then $x = y$.
- If x and y are a pair of alternate interior angles in a transversal of parallel lines, then $x = y$.
- If x and y are a pair of consecutive interior angles in a transversal of parallel lines, then x and y are supplementary.
- If x and y are a pair of alternate exterior angles in a transversal of lines ℓ and m, and $x = y$, then ℓ is parallel to m.
- If x and y are a pair of alternate interior angles in a transversal of lines ℓ and m, and $x = y$, then ℓ is parallel to m.
- If x and y are a pair of consecutive interior angles in a transversal of lines ℓ and m, and x and y are supplementary, then ℓ is parallel to m.

We proved the first three theorems in class; you should be able to prove the last three.

4. Now we can prove that the sum of the interior angles in every triangle is 180 degrees. Let ΔABC be a triangle. Choose, say, $\overline{AB}$ to be the base. Construct line ℓ through C parallel to the base $\overline{AB}$. Extend all three sides of the triangle to lines.

Since $\overleftrightarrow{AC}$ is a transversal to parallel lines ℓ and $\overleftrightarrow{AB}$, then alternate interior angles x and v are equal. Since $\overleftrightarrow{BC}$ is a transversal to parallel lines ℓ and $\overleftrightarrow{AB}$, then alternate interior angles y and w are equal. Since angles v, z, w together form a straight angle, then $v + w + z = 180$ degrees. Therefore $x + y + z = 180$ degrees.

5. We finished class by playing the Quadrilateral Game on pages 74–75 using geoboards.

8 Tuesday, September 22

1. Briefly discussed the Triangle Inequality: Given three positive numbers a, b, c. A triangle exists with these three side lengths if and only if $a + b > c$, $a + c > b$, and $b + c > a$.

2. There is a similar result for quadrilaterals: Given four positive numbers a, b, c, d. A quadrilateral exists with these four side lengths if and only if the sum of every three numbers exceeds the fourth.

9 Thursday, September 24

Exam #1.

10 Tuesday, September 29

1. Used proportional reasoning to estimate the height of individuals from photos of them holding rulers. We noted that measurement inaccuracies lead to inaccurate answers.

2. Made a simple two-dimensional shape using Polydron, and then made a second one similar to the first with a scale factor of two. We say that all lengths doubled, all areas quadrupled, but all angle measures remained the same.

3. Drew simple figures with at least one curved line on half-inch graph paper. We then made a similar figure with scale factor of 2 on one-inch graph paper, and another similar figure with scale factor 1/2 on quarter inch graph paper.

4. Note that the book describes how to make a similar figure with a scale factor of 2 using two rubber bands.

11 Thursday, October 1

1. Looked at three ways to visualize different scales of distance (links are on the course website): Cosmic Scale, The Scale of the Universe, and Cosmic Eye.

2. Followed up on the graph paper figure figures from last time, this time with centimeter paper. Discussed how to determine the scale factors, and also saw that if the scale factor from Figure A to Figure B is s, and the scale factor from Figure B to Figure C is t, then the scale factor from A to C is st.

3. Discussed important concepts related to similar figures. Suppose Figure A is similar to figure B with scale factor t from A to B. By definition, this means that for every distance or length a in A and the corresponding distance or length b in B, the ratio b/a always equals t. So if length p in A corresponds to length q in B, and length r in A corresponds to length s in B, we must have $b/a = q/p = s/r = t$. Notice that we can also determine for such similar figures that $bp = aq$ and so $a/p = b/q$. These ratios are ratios taken *within* each figure, and do not have to equal the scale factor. So, for example, the ratio of a door height to a door width might be 4/1 in both the model of a house and in the actual house, though the actual house may not be related to the model house by a scale factor of 4.

4. Be careful of units of distance and area when considering similar figures. For example, if a map scale is given as 1 inch equals 500 miles, the scale factor from the map to the real world is
$$\frac{500\,\text{mi}}{1\,\text{in}} \times \frac{5280\,\text{ft}}{1\,\text{mi}} \times \frac{12\,\text{in}}{1\,\text{ft}} = 31,680,000.$$
If you are estimating area from the map, and you see that the area is 3 square inches on the map, then the actual area is $3 \times (500)^2\,\text{mi}^2 = 750,000\,\text{mi}^2$.

5. We worked on Problem 2.2.

12 Tuesday, October 6

1. I decided to delay the last problem on the homework due today on mappings of the form $(x, y) \longrightarrow (ax + b, cy + d)$, fix the typos, and will reassign it as part of the next homework. But we also thoroughly discussed this problem in class. We noted that if $|a| = |c|$ and both a and c are nonzero, then the application of such a mapping to a figure results in a similar figure with scale factor $|a|$. The numbers a and c induce scalings in the x and y directions, while the numbers b and d induce translations in the x and y directions.

2. Demonstrated how to use GeoGebra to model mappings like $(x, y) \longrightarrow (ax+b, cy+d)$.

3. Mentioned that a map of the form $(x, y) \longrightarrow (ax, ay)$ with positive a is called a *dilation* with scale factor a and the origin as the center of dilation.

4. Worked on the problem of starting with a three-dimensional "building" made with Multilink cube and creating a similar building with scale factors of 2 and 3. We saw that the measures of angles did not change, the lengths were multiplied by the scale factor, the surface areas were multiplied by the square of the scale factor, and the volumes were multiplied by the cube of the scale factor.

5. Mentioned an application of the above understandings to biology. Volume is associated with weight and heat production, area is associated with strength and heat escape, etc. See the essay "On Being the Right Size," posted to the course website.

13 Thursday, October 8

1. Discussed the effect of the mapping $(x, y) \longrightarrow (-y, x)$ and proved that it causes a rotation by 90 degrees counterclockwise around the origin. We verified that if an arbitrary point P maps to a point P', then $\overline{OP'}$ is perpendicular to $\overline{OP}$, and also that the lengths OP and OP' are the same.

2. Worked on selected homework problems.

14 Tuesday, October 13

Volume and surface area of rectangular prisms.

15 Thursday, October 15

Volume and surface area of polygonal prisms.

16 Tuesday, October 20

Circumference and area of circles

17 Thursday, October 22

Circumference and area of circles

18 Tuesday, October 27

1. How to make drawings on isometric graph paper, and example of an app from Illuminations. There is also a link to a nice iPad app on the webpage.

2. Reminder of what ratios give the scale factor in similar figures.

3. All circles are similar, so $C/c = D/d$ for any two circles, the first with circumference c and diameter d, and the second with circumference C and diameter D. Rearranging, we get $c/d = C/D$ for any two circles, and this common ratio is defined to be the number π. Thus $C = \pi D$, or as it is more commonly written, $C = \pi d = 2\pi r$ for a circle of radius r, diameter d, and circumference C.

4. π can be approximated by inscribing regular polygons in a circle and calculating their perimeters.

5. The area of a circle $A = \pi r^2$ can be motivated by "unrolling" concentric circles, or by cutting up the circle into pie-shaped pieces and rearranging them.

19 Thursday, October 29

1. Recalled volume and surface area for rectangular prisms with dimensions a, b, c:

$$V = abc$$

and

$$S = 2ab + 2ac + 2bc$$

2. Discussed volume and surface area for (right) prism having bases that are polygons. If B is the area of the base, P is the perimeter of the base, and h is the height, then

$$V = Bh$$

and

$$S = 2B + Ph.$$

3. Discussed volume and surface area for pyramids having bases that are polygons. If B is the area of the base and h is the height, then

$$V = \frac{1}{3}Bh.$$

We motivated this by constructing three particular pyramids that fit together to form a cube. As for surface area,

$$S = B + \text{the areas of the various lateral triangles.}$$

In the special case that the base is a regular polygon with the apex of the pyramid centered over the base, then each lateral triangle is isosceles and congruent to each other, and

$$S = B + \frac{1}{2}P\ell,$$

where P is the perimeter of the polygon and ℓ is the slant height of each triangle.

4. Note that SketchUp is very useful in making drawings of these objects.

5. Watched four Vi Hart YouTube videos: Optimal Potatoes, Green Bean Matherole, Borromean Onion Rings, Thanksgiving Turduckenen-duckenen.

6. Homework: Construct a cone and bring to class on Tuesday.

20 Tuesday, November 3

1. Developed the formulas for a cylinder of radius r and height h:

$$V = \pi r^2 h,$$

$$S = 2\pi r^2 + 2\pi r h.$$

   We got this surface area formula by unwrapping.

2. For a cone of radius r and height h, we stated, without proof, that the volume is given by

$$V = \frac{1}{3}\pi r^2 h.$$

   This is analogous to the formula for the volume of a pyramid. But we developed the surface area formula in two different ways: one by unwrapping, and one by approximations with skinny triangles.

$$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}.$$

3. For a sphere of radius r, we stated, without proof, that the volume is given by

$$V = \frac{4}{3}\pi r^3.$$

   Given this, we derived the formula relating volume and surface area by approximating the solid sphere with a dissection into skinny pyramids, getting $V = \frac{1}{3}rS$. From this we derived

$$S = 4\pi r^2.$$

4. We noted that the derivative of the formula for the volume of a sphere equals the formula for the surface area of a sphere, and also that the derivative for the formula for the area of a circle equals the formula for the circumference of a circle, and why this makes sense.

5. With the help of Desmos, determined the proportions of a cylinder of given volume that has minimum surface area.

21 Thursday, November 5

1. Answered homework questions.

2. Discussed some topics in triangle similarity. In particular, two triangles are similar if and only if there is a correspondence of vertices in which corresponding angles have the same measure. From this we can see that two right angles are similar if and only if one of the acute angles has the same measure in both triangles. And then from this we can see that if two right triangles are congruent, then for the matching acute angles the length of the opposite side divided by the length of the hypotenuse matches in both triangles. Thus we can make sense of giving this ratio a name—in this case, sine.

22 Tuesday, November 10

1. Answered questions relating to the upcoming exam.
2. Mentioned some interesting facts about the occurrences of the golden ratio and Fibonacci numbers in art and nature.

23 Thursday, November 12

Exam #2.

24 Tuesday, November 17

1. Constructed squares of various areas on a grid (a problem from the text). Not all whole number areas are obtainable—we saw that the area had to be expressible as the sum of two perfect squares. This problem provides some motivation for the Pythagorean Theorem.

2. Discussed two proofs of the Pythagorean Theorem, one via a puzzle (from the text), and the other via similar triangles (from the exam).

25 Thursday, November 19

1. Discussed the definitions and properties of rational numbers irrational numbers.

2. Started working on homework problems.

26 Tuesday, November 24

Discussed rotations, reflections, and translations from the first chapter of *Butterflies, Pinwheels, and Wallpaper.*

27 Thursday, November 26

Thanksgiving—no class.

28 Tuesday, December 1

1. Reviewed three transformations: translations, reflections, and rotations, and what elements are needed to define each one.

2. Practiced using rulers and protractors to apply these transformations to various figures. Developed instructions for doing each one.

3. Worked on problems in which we needed to identify the transformation involved. Determined how to find the elements of each. Also did this with the GeoGebra files on the course website.

29 Thursday, December 3

Worked on homework.

30 Tuesday, December 8

1. Defined congruence via transformations and discussed various criteria for triangle congruence.

2. Worked on homework.

31 Thursday, December 10

1. Proved that opposite sides of a parallelogram are congruent.

2. Worked on homework.