MA 308 Course Notes

Carl W. Lee

Spring 2011

Contents

1 Thursday, January 13

1. Room Diagonal Problem. Working in groups, find the length of a main diagonal of the classroom. Meter sticks were provided. Solutions were discussed. Some outcomes:

   (a) There were different solutions—one using the three-dimensional distance formula, the others using two applications of the Pythagorean Theorem. It was recognized that all four main diagonals of the room would have the same length.

   (b) Mathematical elements that arose in the course of this problem included

   - Coordinate systems. Standard coordinate system in two dimensions. Standard "right-handed" coordinate system in three dimensions.
   - The distance formula in two and three dimensions. Two dimensions: The distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Three dimensions: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
   - The Pythagorean Theorem.
   - Measurement and accuracy. What are the effects of our measuring tools?
   - Choosing units (meters or centimeters?).
   - Visualization of three dimensional objects.
   - Representations with drawings.
   - Nets—unfoldings of three-dimensional shapes.
   - Using algebra as a shortcut. Example: We could solve the problem in two steps: $\ell_1 = \sqrt{L^2 + W^2}$ then $\ell_2 = \sqrt{\ell_1^2 + H^2}$, or we could then see that we could solve it in one step: $\ell_2 = \sqrt{L^2 + W^2 + H^2}$. In the latter case we also see the relationship to the three-dimension distance formula.

2. We reflected on the learning experience—How do people learn? What are different learning styles (reading/writing, aural, visual, kinesthetic, etc.)? Why must a teacher be aware of potential solutions and explanations other than his/her own? How can a teacher avoid the trap of checklists and facilitate strong connection making by careful selection of activities and classroom experiences?

3. While you are engaging in this course as students, you must simultaneously be reflecting as a future teacher.

4. Passed out the syllabus.

5. One main point of this course is to appreciate this two way street: We can use our knowledge and understanding to solve problems, and by solving problems we can increase our knowledge and understanding of mathematics.

6. Showed one of the classrooms produced by students at Jessie Clark Middle School. More can be found at the website `http://www.ms.uky.edu/~lee/jessieclark/jessieclark.html`.

2 Tuesday, January 18

1. Introduced the Mathematics Education and Outreach Resources list from course website, `http://www.ms.uky.edu/~lee/outreach/outreach.html`.

2. Introduced the Common Core State Standards document, `http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf`.

3. Gave a quick demo of Google SketchUp, showing how to construct and measure a room diagonal, `http://sketchup.google.com`.

4. Announced the first homework assignment, which will be emailed out and posted on the course website.

5. Round of introductions, including a significant or memorable classroom mathematical learning moment.

6. Began working on the Locker Problem.

   (a) Middle school lesson: Investigation 5 from *Connected Mathematics, Grade Six, Prime Time*. Handout.

   (b) Website for the Connected Mathematics Project: `http://connectedmath.msu.edu`.

   (c) Technology: Locker Problem applet for Prime Time: `http://connectedmath.msu.edu/CD/Grade6/Locker/index.html`.

3 Thursday, January 20

1. Continued to discuss the Locker Problem.

   (a) Discussed how to obtain a prime factorization of a positive integer by means of a factor tree. Technology: Factor Tree applet, from the National Library of Virtual Manipulatives, `http://nlvm.usu.edu`.

   (b) Discussed how to use prime factorizations to:

   - Obtain a formula for the number of factors, by using the Multiplication Counting Principle to count the result of making sequential choices.
   - Systematically list all of the factors by means of a decision tree.
   - Find the greatest common factor and least common multiple of two positive integers.

   (c) Characterized which positive integers have exactly 2 factors, exactly 3 factors, and exactly 4 factors.

   (d) Noted the mathematical concepts that were associated with working on this problem:

   - Factors (divisors)
   - Multiples
   - Perfect squares
   - Factor tree
   - Prime factorization
   - Decision tree (and the multiplication counting principle)
   - Prime numbers
   - Common factors and greatest common factor
   - Common multiples and least common multiple

2. Considered the questions: What is 12 divided by 4 and why does your answer make sense? What is 12 divided by $\frac{1}{2}$ and why does your answer make sense?

3. Watched the video on Defending Reasonableness: Division of Fractions, from the book *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning* by Jo Boaler and Cathy Humphreys. Handout: Transcript and discussion questions.

4 Tuesday, January 25

1. Worked on some problems about making sense of division with fractions. Handout: Dividing Fractions.

2. Announced that Professor Ben Braun will be teaching the class on Thursday.

3. Technology: Introduced the powerful computational website WolframAlpha, `http://www.wolframalpha.com`.

4. Discussed the questions associated with the video (handout from last class).

5. Watched the video on the interview with some of the middle school students; discussed the role of problem-solving in long-lasting learning.

6. Collected Homework #1, with a very brief discussion of some of the Common Core State Standards selected by class members.

7. Homework #2 will be sent out and posted later today.

5 Thursday, January 27

1. Handout of Teaching Beliefs, by Cathy Humphreys.
2. Worked on the Crossing the River problem. Handout.

6 Tuesday, February 1

1. Passed back Homework #1. Discussed how to derive the distance formula from the Pythagorean Theorem. Emphasized the goal of understanding and being able to articulate: (1) why perfect squares have an odd number of factors (and why nonperfect squares don't), (2) why the formula for the number of factors of a positive integer using the exponents in its prime factorization works, and (3) how and why you can systematically list all of the factors of a positive integer by knowing its prime factorization.

2. Collected Homework #2. The assignment for Homework #3 will be sent out soon.

3. Technology: Demonstrated the program GeoGebra, which can be used for geometric sketching and algebraic plotting, `http://www.geogebra.org`. Demonstrated Jing, which can be used to make screenshots and videos of your computer screen, `http://www.techsmith.com/jing`. Mentioned the program Dropbox, which can be used to share files across computers, `http://www.dropbox.com`—2GB of storage is free.

4. Discussed the solution to the coin weighing problem from Homework #2. This problem addresses the algebraic practice of algorithmic thinking. We extended this problem to more than two weighings, seeing a systematic way to detect the heavy counterfeit coin among 3^k coins in k weighings.

5. Working on the problem of 1's and 2's—how many ways can you write the positive integer n as a sum of 1's and 2's, when order matters? We saw that this produces the sequence of the Fibonacci numbers, and explained why this result is true by getting all the sums for n from all the sums for $n-1$ and for $n-2$. This problem is on the handout Adding it Up, which will be distributed next time.

7 Thursday, February 3

1. Exam reminder: February 10. I will prepare some study guidance, which will be sent out later.

2. Handout for your notebooks: Adding it Up (since we were considering some of these problems).

3. Three Algebraic Thinking Habits of Mind. Refer to the Algebra book, pp. 1–2. Handout: Three Problems.

   - Doing-Undoing. Example: How many factors does the number 60 have? Answer: $60 = 2^2 \cdot 3 \cdot 5$, so the number of factors is $(2+1)(1+1)(1+1) = 12$. Find a positive integer that has 2 prime factors and a total of 15 factors. There are many answers—construct numbers of the form p^2q^4, where p and q are both prime.
   - Building Rules to Represent Functions. Rules may be recursive or explicit. Example: Guess a rule for the maximum number of regions you can divide the plane into with n lines. Conjectured recursive formula: $f(1) = 2$ and thereafter $f(n) = f(n-1) + n$. Conjectured explicit formula: $f(n) = \frac{n(n+1)}{2} + 1$.
   - Abstracting from Computation. Example: Write down three consecutive integers a, b, c. Compare $a \times c$ with b^2. Repeat several times. Explain. We observed that it appears that $ac = b^2 - 1$ or $b^2 = ac + 1$. We saw two explanations. Looking at a specific example, we saw $5 \times 5 = (4 \times 5) + 5 = (4 \times 6) - 4 + 5 = (4 \times 6) + 1$. By algebra, using the consecutive numbers $a, a+1, a+2$, we saw $a(a+2) = a^2 + 2a$ and $(a+1)^2 = a^2 + 2a + 1$. Alternatively, representing the consecutive numbers as $a-1, a, a+1$, we saw $(a-1)(a+1) = a^2 - 1$. We noted that this holds for all real numbers.

4. As a follow-up, we discussed two ways to obtain the formula for $1+2+3+\cdots+n$—one with a diagram of an $n \times (n+1)$ rectangle, and the other with adding $S = 1+2+3+\cdots+n$ to $S = n + (n-1) + (n-2) + \cdots + 1$.

5. Worked on the Towers of Hanoi problem with playing cards. This problem is on the National Library of Virtual Manipulatives, `http://nlvm.usu.edu`. Go to Algebra 6–8, and then select Towers of Hanoi. We came up with two recursive formulas and one explicit formula. Some groups came up with an explanation of why one of the recursive formulas makes sense.

8 Tuesday, February 8

1. Discussed how to represent and understand fraction multiplication with rectangles. Website: National Library of Virtual Manipulatives, `http://nlvm.usu.edu`; select Number and Operations 3–5, and then Fractions - Rectangle Multiplication.

2. Discussed solutions to Homework #3. This assignment will be collected next time.

3. Answered questions associated with the exam review.

4. A new homework assignment will be sent out soon.

9 Thursday, February 10

Exam #1.

10 Tuesday, February 15

1. Discussed the Animal Problem, and how the algorithmic aspects of this problem can lead to an algebraic analysis of what is going on.

2. Returned exams and homework.

3. Technology: Demonstrated how to use spreadsheets for formulas defined recursively and explicitly. In particular, typing "`Ctrl-~`" toggles the view between the results of the formulas, and the actual formulas in the various cells.

4. Discussed Homework #4, problem #1, the chocolate problem. We saw four different approaches to this problem, with different representations of the the solutions. Some additional assumptions had to be made. For example, we might assume that Pam ate the same number of chocolates on each of the two days, or we might assume that a whole number of chocolates must be eaten by each person on each day. This is an example of one of the resources available from NCTM — The National Council of Teachers of Mathematics — `http://www.nctm.org`. I strongly encourage everyone to join! In addition to lessons and teacher resources, it also has a page on Facebook, which is where I found this particular problem.

5. Discussed Homework #4, problem #4, the calendar cube problem. Here, it is necessary to think outside the box, and realize that you can invert the 6 to get the 9. This is an example of one of the Puzzlers from Car Talk, `http://www.cartalk.com/content/puzzler`.

6. Discussed Homework #4, problem #2, the crawling snail problem. The challenge here seemed to be to find a concise way to describe the discovered pattern.

7. Briefly discussed two problems related to the previous one in the sense that you have to "think outside the box".

   (a) Suppose you are in a room in which there are two long pieces of string hanging from the ceiling. They are far enough apart so that if you hold onto the end of one, you are unable to reach the other. However, they are long enough so that if you could bring the two hanging ends toward each other, you would be able to tie them together. You are alone, however, and all you have is a pair of scissors. Without cutting the strings or removing them from their locations, how can you manage to tie the hanging ends together?

(b) You have six toothpicks, each 3 inches long. How can you use these to form 4 equilateral triangles, each having side length 3?

8. Collected homework.

9. Homework assignment #5 was sent out afterward by email.

11 Thursday, February 17

1. Briefly discussed another problem from Car Talk: the three prisoners and the five hats. See `http://cartalk.com/content/puzzler/transcripts/201048/index.html`.

2. Handout: CMP's *Say it With Symbols*, Investigation 1.

3. Worked on and discussed the Tiling Pool Problem, which is Problem 1.1 on page 6 of CMP's *Say it With Symbols*. This is a good problem to illustrate the principle of Abstracting from Computation, and draws students into thinking about equivalent expressions.

4. Brief reminders of important properties of real numbers:

   (a) Commutative property of addition.

   (b) Commutative property of multiplication.

   (c) Associative property of addition.

   (d) Associative property of multiplication.

   (e) Additive identity element.

   (f) Multiplicative identity element.

   (g) Additive inverses.

   (h) Multiplicative inverses.

   (i) Distributive property of addition over multiplication.

5. Worked on and discussed the Community Pool Problem, which is Problem 1.3 on pages 8–9 of CMP's *Say it With Symbols*.

6. Looked at the rectangle representation of the Distributive Property. We saw the related applet Algebra Tiles on the National Library of Virtual Manipulatives, `http://nlvm.usu.edu`. Go to to Algebra, Grades 6–8. This applet can be also used to visualize and practice the factoring of quadratics.

12 Tuesday, February 22

1. Discussed the second formula of #58 on page 21 of *Say it With Symbols*.

2. Announced that I will have a substitute for class on Tuesday, March 1 (it will be Professor Braun).

3. Briefly discussed the solution to the Painted Cube problem on the homework.

4. Reviewed some types of functions:

   (a) Linear. A function of the form $y = mx + b$.

   (b) Quadratic. A function of the form $y = ax^2 + bx + c$, where a is not equal to zero.

   (c) Exponential. A function of the form $y = a \cdot b^x$, where a and b are constants and b is positive.

   (d) Proportional. y is proportional to x if if there is a nonzero constant k such that $y = kx$. That is to say, the ratio $\frac{y}{x}$ is a constant.

   (e) Inversely or indirectly proportional. y is inversely proportional to x if y is proportional to $\frac{1}{x}$. That is to say, there is a nonzero constant k such that $y = \frac{k}{x}$, or yx is a constant.

   (f) We can define functions this way: A relation is a set of ordered pairs. A ficklepicker in a relation is a first coordinate that appears in more than one of the ordered pairs. A function is a relation with no ficklepickers.

5. The next homework assignment will be sent out after class on Thursday.

13 Thursday, February 24

1. Warm-Up Problem: Write a quadratic equation whose solutions are $x = -5$ and $x = 3$.

2. Briefly discussed the "four-fold way" of describing or representing functions:

   (a) Verbally (orally, or in writing).

   (b) Algebraically.

   (c) Graphically.

   (d) Numerically (e.g., with a table of values).

   Mentioned that if y is a linear function of x, it is not always the case that y is proportional to x, but it is the case (for a nonconstant function) that Δy is proportional to Δx. This is one way to recognize a linear function from a table of values. The constant ratio $\frac{\Delta y}{\Delta x}$ is the slope of the line that is the graph of the function.

3. Returned homework.

4. Mentioned the Microsoft Mathematics module for Microsoft Word, `http://www.microsoft.com/downloads/en/details.aspx?FamilyID=9caca722-5235-401c-8d3f-9e242b794c3a`. I would be very grateful if someone with a Windows operating system would download and demonstrate this.

5. Discussed *Say it With Symbols*, p. 30, #11, except we assumed we had 242 meters of fence. There are several approaches to this problem. We saw that we could use the formula for perimeter to solve for ℓ in terms of w, and then substitute into the formula for area. The resulting expression for A in terms of w is a quadratic, its graph is a parabola opening downward, and by symmetry the value of w for its high point (vertex) is halfway between the w-intercepts. The vertical line through this point is a line of symmetry, and is the axis of the parabola. It was observed that for a quadratic function $y = ax^2 + bx + c$, the vertex of the parabola occurs when $x = -\frac{b}{2a}$. This makes sense from the quadratic formula—in the case that there are x-intercepts, the two roots are

$$-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \text{ and } -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}.$$

   So the axis of the parabola crosses the x-axis exactly in between. We also solved the fencing problem using calculus, and using WolframAlpha.

6. Started the Soda Can problem. Handout. We reviewed why the formulas for surface area and volume made sense. We noted that 1 mL (milliliter) equals 1 cm^3 (cubic centimeter). There are similarities in approach with the fencing problem.

7. Homework #6 will be sent out today, due in one week.

14 Tuesday, March 1

1. Worked on *Say it With Symbols*, p. 48, #25.
2. Worked on the problem of creating a model of a cube to represent the equation $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$.
3. Began working on the *Say it With Symbols*, p. 58, Problem 4.2, parts A, B, and C.

15 Thursday, March 3

1. Reviewed the solutions to Homework #6, collected today.

   (a) Discussed the solution to the soda can problem, including ways to approach this problem without calculus; e.g., using graphing calculators, GeoGebra, or WolframAlpha for making graphs. It is strongly recommended that you become comfortable with using the graphing calculator for graphing and analyzing functions.

   (b) Discussed at one dissection to model the equation $x^2 - 9 = (x-3)(x+3)$.

   (c) Discussed *Say it With Symbols*, p. 58, Problem 4.2, parts A, B, and C1. In particular, we saw that for the linear and exponential functions there was only one solution for each—only one way to extend the table and to obtain a formula—whereas for the quadratic function there were an infinite number of choices.

      i. In tables of linear functions, of the form $y = mx + b$, as x increases by fixed increments, say, $x = 1, 2, 3, \ldots$, then y also must change by adding fixed increments. (If x increases by 1, then y changes by adding m.) We discussed how this was related to the slope of the graph of the function, and also to its equation. Once you know two entries in the table or two points in the graph, the function is completely determined.

      ii. In tables of exponential functions, of the form $y = a \cdot b^x$, as x increases by fixed increments, say, $x = 1, 2, 3, \ldots$, then y changes by being multiplied by a fixed amount. Again, once you know two entries in the table or two points in the graph, the function is completely determined. (If x increases by 1, then y changes by multiplying by b.)

      iii. In tables of quadratic functions, of the form $y = ax^2+bx+c$, as x goes through the consecutive values $1, 2, 3, \ldots$, the values of y do not increase by fixed amounts, but the *increases* increase by fixed amounts—here we are looking at "second differences". In general you need three entries in the table or three points on the graph to pin the function down. You can substitute points into the equation of the function and solve three equations simultaneously to find a, b, and c.

      iv. If you have a polynomial function of degree k, this can be detected by taking differences of differences of differences, etc., of the y values for consecutive x values. Doing this k times will result in a constant kth difference.

   (d) Discussed the solution to problem #5, given the above understandings.

2. Discussed some motivations for rules for the sign of the product of two integers: mail carriers delivering or taking away a certain number of checks or bills; moving right and left into the future or into the past; running water into or draining it out of a bathtub, while time runs forward or backward, etc.

3. Reminder that Exam #2 will take place in one week. I will send out some review notes.

16 Tuesday, March 8

1. Reviewed for Exam #2 on Thursday.

2. Briefly described how to use a TI calculator to graph a function and find a minimum value.

3. Mentioned that the formula for simple interest, $A = P(1 + rt)$, is a linear function of the time t. Note that P is the initial amount (principal), r is the annual interest rate expressed as a decimal, and t is the time span in years. On the other hand, the formula for compound interest (compounded annually), $A = P(1+r)^t$, is an exponential function of the time t.

17 Thursday, March 10

Exam #2.

18 Tuesday, March 22

1. Warm-up problem on proportional reasoning: If a chicken and a half can lay an egg and a half in a day and a half, how long will it take 12 chickens to lay 60 egg? We solved this by recognizing that the number of eggs E is proportional to the number of chickens C and the number of days D. (So C and D are inversely proportional.) We can use $E = kCD$, solve for k from the given data, and then solve for D from the new data.

2. Returned exams and distributed grade reports.

3. iMovie example. Demonstrated a simple movie made with still images and a sound track.

4. Worked on the the Guitar Fret Problem.

19 Thursday, March 24

1. Exam #3 will NOT be on April 7 – this is coming up too soon. We will aim for April 14 instead.

2. Introduced the website Interactivate, which has a substantial number of lessons and activities keyed to the Common Core Standards, `http://www.shodor.org/interactivate`.

3. Passed out Homework #7. I strongly recommend trying to get Google Sketchup working before next class, in case there are any problems.

4. Worked on the Guitar Fret Problem—see the handout.

5. Demonstrated the virtual piano, `http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/music/piano/index.htm`. Pairs of notes that are played together as a chord tend to sound more compatible when the fractions expressing the ratios of their frequencies involve smaller integers.

6. I mentioned the existence of LaTeX for mathematical typesetting. It can be downloaded for free, but takes some time to get used to it. Here is a website: `http://www.latex-project.org`.

20 Tuesday, March 29

1. Watched the video on Infinity Elephants by Vi Hart, `http://www.youtube.com/watch?v=DK5Z709J2eo`. This relates to the guitar fret problem in that in theory there would be an infinite sequence of frets on the guitar as you approach the bridge. Also, it shows the power of combining mathematical topics with media that are becoming more and more available in schools.

2. Watched a video on 3d printers, another example of technology that is working its way into schools, and can be combined with geometry for some applications and enhanced understanding. `http://www.youtube.com/watch?v=Jxt3w3FXWrc`.

3. Demonstrated some features of SketchUp. Some class members were having problems downloading and installing it — please keep trying, or else partner with someone who has been successful.

4. Point and Segment Rotation Problems — handout. Worked on these two problems and discussed the solutions, including proofs that (1) if a point is on the perpendicular bisector of the line segment joining the two given points, then it is equidistant from the two points, and (2) if a point is equidistant from the two given points, then it is on the perpendicular bisector of the line segment joining the two points. Also mentioned the distinction between rotating clockwise and counterclockwise, and the possibility of rotating more than 360 degrees. Used GeoGebra as a tool in the discussion.

21 Thursday, March 31

1. Collected Homework #7.

2. Homework #8 was assigned after class.

3. Demonstrated the use of GeoGeobra to export to a webpage applet.

4. Gave an example of a lesson from NCTM's Illuminations website—fretted musical instruments, `http://illuminations.nctm.org/LessonDetail.aspx?ID=U163`. It is worth joining NCTM for the immense amount of support and resources throughout your teaching career!

5. Worked on another center of rotation problem, this time with two lines segments that were not oriented perpendicular to each other. Learned that the center of rotation can be found be intersecting the perpendicular bisectors of the line segments joining pairs of corresponding points. In this case there were two centers of rotation.

6. Watched a portion of the video clip included with your text book that relates to the above problem—see pages 65ff in your book.

7. Looked at examples of the theorem that if you place two congruent shapes (in this case, the letter "F") in the plane with the same orientations (i.e., both "face up"), then the two shapes are related by either a translation or a rotation.

8. Began analyzing symmetry patterns—handout. See `http://www.ms.uky.edu/~lee/ma111fa09/slides11.pdf`.

22 Tuesday, April 5

1. Excerpt from The Hobart Shakespeareans illustrating power of activiting engagement.
2. Brief look at symmetries of patterns.
3. Work on homework — creating patterns.

23 Thursday, April 7

1. Discussed and collected homework.

2. Mentioned four theorems about concurrent lines in a triangle.

   (a) The three perpendicular bisectors meet in a common point.

   (b) The lines containing the three altitudes meet in a common point.

   (c) The three medians meet in a common point, and this point of intersection divides each of them in a ratio of 2:1.

   (d) The three angle bisectors meet in a common point.

3. Mentioned that there are 17 different types of wallpaper patterns (according to their symmetries), and that any particular pattern can be analyzed using, for example, a flowchart. See `http://mathcs.slu.edu/escher/index.php/Wallpaper_Patterns#Wallpaper_Flow_Chart`.

4. Worked on problem of composing motions. In particular, used physical and GeoGebra models to illustrate and understand that a reflection in a line followed by a reflection in a parallel line results in a translation, and a reflection in a line followed by a reflection in an intersecting line results in a rotation.

5. Mentioned, but did not carry out, the activity of investigating the compositions of the symmetries of a square with a physical activity.

6. Reminder that Exam #3 will be Thursday, April 14. I will not be there and will arrange for a proctor.

24 Tuesday, April 12

1. Practiced with Miras for constructing and analyzing reflections.

2. Questions and answers relating to the material for the exam. Practiced with some wallpaper patterns, http://www.ms.uky.edu/~lee/ma111fa09/patterns.pdf.

3. Viewed video on the Visible Human Project — cross-sections of a human being, http://www.youtube.com/watch?v=iWP2HnPSMyo.

4. Demonstrated making cross-sections with SketchUp.

5. Gave false proof that all triangles are isosceles — see http://en.wikipedia.org/wiki/Mathematical_fallacy#Fallacy_of_the_isosceles_triangle and video at http://www.metacafe.com/watch/418734/all_triangles_are_isosceles.

25 Thursday, April 14

Exam #3

26 Tuesday, April 19

1. Looked at some ambigram creations of Scott Kim, `http://scottkim.com/thinkinggames/nctm2011handout.pdf`. His website: `scottkim.com`. From his talk at NCTM, with a great set of links: `http://scottkim.com/thinkinggames/nctm2011.html`.

2. Worked on a scaling problem: Build a structure using four multilink cubes, then build another structure that is similar but with a scaling factor of three. Carefully explain why linear measurements, such as lengths and perimeters, scale up by a factor of 3; surface areas scale up by a factor of 9; and volumes scale up by a factor of 27. Discuss how these principles apply to more general three dimensional objects. One way to illustrate: Lego structures (lots of images on the web).

3. Mentioned the Soma Cube puzzle. Here is one link: `http://en.wikipedia.org/wiki/Soma_cube`.

4. If an 8 inch (diameter) pizza costs \$8, and a 12 inch pizza costs \$12, which one is the better bargain? (Should we consider area? Do pizzerias use the same amount of dough and ingredients in both but just spread things out more?)

5. Mentioned the Geoboard as a useful classroom tool for some geometry concepts, including scaling of plane figures. There is also an applet at the National Library of Virtual Manipulatives, `http://nlvm.usu.edu/en/nav/vlibrary.html`.

6. Began working on some probability problems. Handout: `http://www.ms.uky.edu/~lee/ckpims09/notes.pdf`—see Day Three. Discussed Dice Differences, in which two players each roll a die. If the absolute value of the difference is 0, 1, or 2, the first player wins. If it is 3, 4 or 5, the second player wins.

27 Thursday, April 21

1. Worked on probability problems from `http://www.ms.uky.edu/~lee/ckpims09/notes.pdf`: The Second Girl (an example of conditional probability), Strange Dice (an example of a possibly counterintuitive result), the Gambler's Scheme (which seems like a sure win, but the finite amount of money available foils the plan), Ill or Not (another examople of conditional probability).

28 Tuesday, April 26

1. Clarified how to calculate expcted value, revisiting the Gambler's Scheme Problem in the case that the gambler had $15 to start with.

2. Discussed probability problems from `http://www.ms.uky.edu/~lee/ckpims09/notes.pdf`: What Should We Expect Next? and Expectations Aren't Everything.

3. Discussed examples from the course of the Mathematical Practices of the Common Core State Standards—see pages 6–8 of `http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf`.

4. Homework #9 will be 50 points given for the class discussion of the probability problems.