Communication

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Communication is an essential part of mathematics and mathematics education. It is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing. Listening to others' explanations gives students opportunities to develop their own understandings. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. Students who are involved in discussions in which they justify solutions—especially in the face of disagreement—will gain better mathematical understanding as they work to convince their peers about differing points of view (Hatano and Inagaki 1991). Such activity also helps students develop a language for expressing mathematical ideas and an appreciation of the need for precision in

that language. Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically.

Because mathematics is so often conveyed in symbols, oral and written communication about mathematical ideas is not always recognized as an important part of mathematics education. Students do not necessarily talk about mathematics naturally; teachers need to help them learn how to do so (Cobb, Wood, and Yackel 1994). As students progress through the grades, the mathematics about which they communicate should become more complex and abstract. Students' repertoire of tools and ways of communicating, as well as the mathematical reasoning that supports their communication, should become increasingly sophisticated. Support for students is vital. Students whose primary language is not English may need some additional support in order to benefit from communication-rich mathematics classes, but they can participate fully if classroom activities are appropriately structured (Silver, Smith, and Nelson 1995).

Students need to work with mathematical tasks that are worthwhile topics of discussion. Procedural tasks for which students are expected to have well-developed algorithmic approaches are usually not good candidates for such discourse. Interesting problems that "go somewhere" mathematically can often be catalysts for rich conversations. Technology is another good basis for communication. As students generate and examine numbers or objects on the calculator or computer screen, they have a common (and often easily modifiable) referent for their discussion of mathematical ideas.

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Organize and consolidate their mathematical thinking through communication

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Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to a » classmate or teacher, or

when they formulate a question about something that is puzzling to them. Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson (Silver, Kilpatrick, and Schlesinger 1990).

Reflection and communication are intertwined processes in mathematics learning. With explicit attention and planning by teachers, communication for the purposes of reflection can become a natural part of mathematics learning. Children in the early grades, for example, can learn to explain their answers and describe their strategies. Young students can be asked to "think out loud," and thoughtful questions posed by a teacher or classmate can provoke them to reexamine their reasoning. With experience, students will gain proficiency in organizing and recording their thinking.

Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson. Later, they may find it helpful to reread the record of their own thoughts.

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Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

In order for a mathematical result to be recognized as correct, the proposed proof must be accepted by the community of professional mathematicians. Students need opportunities to test their ideas on the basis of shared knowledge in the mathematical community of the classroom to see whether they can be understood and if they are sufficiently convincing. When such ideas are worked out in public, students can profit from being part of the discussion, and the teacher can monitor their learning (Lampert 1990). Learning what is acceptable as evidence in mathematics should be an instructional goal

from prekindergarten through grade 12.

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To support classroom discourse effectively, teachers must build a community in which students will feel free to express their ideas. Students in the lower grades need help from teachers in order to share mathematical ideas with one another in ways that are clear enough for other students to understand. In these grades, learning to see things from other people's perspectives is a challenge for students. Starting in grades 3–5, students should gradually take more responsibility for participating in whole-class discussions and responding to one another directly. They should become better at listening, paraphrasing, questioning, and interpreting others' ideas. For some students, participation in class discussions is a challenge. For example, students in the middle grades are often reluctant to stand out in any way during group interactions. Despite this fact, teachers can succeed in creating communication-rich environments in middle-grades mathematics classrooms. By the time students graduate from high school, they should have internalized standards of dialogue and argument so that they always aim to present clear and complete arguments and work to clarify and complete them » when they fall short. Modeling and carefully posed questions can help clarify age-appropriate expectations for student work.

Written communication should be nurtured in a similar fashion. Students begin school with few writing skills. In the primary grades, they may rely on other means, such as drawing pictures, to communicate. Gradually they will also write words and sentences. In grades 3–5, students can work on sequencing ideas and adding details, and their writing should become more elaborate. In the middle grades, they should become more explicit about basing their writing on a sense of audience and purpose. For some purposes it will be appropriate for students to describe their thinking informally, using ordinary language and sketches, but they should also learn to communicate in more-formal mathematical ways, using conventional mathematical terminology, through the middle grades and into high school. By the end of the high school years, students should be able to write well-constructed

mathematical arguments using formal vocabulary.

Examining and discussing both exemplary and problematic pieces of mathematical writing can be beneficial at all levels. Since written assessments of students' mathematical knowledge are becoming increasingly prevalent, students will need practice responding to typical assessment prompts. The process of learning to write mathematically is similar to that of learning to write in any genre. Practice, with guidance, is important. So is attention to the specifics of mathematical argument, including the use and special meanings of mathematical language and the representations and standards of explanation and proof.

As students practice communication, they should express themselves increasingly clearly and coherently. They should also acquire and recognize conventional mathematical styles of dialogue and argument. Through the grades, their arguments should become more complete and should draw directly on the shared knowledge in the classroom. Over time, students should become more aware of, and responsive to, their audience as they explain their ideas in mathematics class. They should learn to be aware of whether they are convincing and whether others can understand them. As students mature, their communication should reflect an increasing array of ways to justify their procedures and results. In the lower grades, providing empirical evidence or a few examples may be enough. Later, short deductive chains of reasoning based on previously accepted facts should become expected. In the middle grades and high school, explanations should become more mathematically rigorous and students should increasingly state in their supporting arguments the mathematical properties they used.

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Analyze and evaluate the mathematical thinking and strategies of others

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In the process of working on problems with other students, learners gain several benefits. Often, a student who has one way of seeing a problem can profit from

another student's view, which may reveal a different aspect of the problem. For example, students who try to solve the following problem (Krutetskii 1976, p. 121) algebraically often have difficulty setting up the equations, and they benefit from the insights provided by students who approach the problem using visual representations. »

> There are some rabbits and some hutches. If one rabbit is put in each hutch, one rabbit will be left without a place. If two rabbits are put in each hutch, one hutch will remain empty. How many rabbits and how many hutches are there?

It is difficult for students to learn to consider, evaluate, and build on the thinking of others, especially when their peers are still developing their own mathematical understandings. A good setting in which young students can share and analyze one another's strategies is in solving arithmetic problems, where students' invented strategies can become objects of discussion and critique. Students must also learn to question and probe one another's thinking in order to clarify underdeveloped ideas. Moreover, since not all methods have equal merit, students must learn to examine the methods and ideas of others in order to determine their strengths and limitations. By carefully listening to, and thinking about, the claims made by others, students learn to become critical thinkers about mathematics.

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Use the language of mathematics to express mathematical ideas precisely

As students articulate their mathematical understanding in the lower grades, they begin by using everyday, familiar language. This provides a base on which to build a connection to formal mathematical language. Teachers can help students see that some words that are used in everyday language, such as *similar, factor, area,* or *function,* are used in mathematics with different or more-precise meanings. This observation is the foundation for understanding the concept of mathematical definitions. It is important to give students experiences that help them appreciate the power and precision of mathematical

language. Beginning in the middle grades, students should understand the role of mathematical definitions and should use them in mathematical work. Doing so should become pervasive in high school. However, it is important to avoid a premature rush to impose formal mathematical language; students need to develop an appreciation of the need for precise definitions and for the communicative power of conventional mathematical terms by first communicating in their own words. Allowing students to grapple with their ideas and develop their own informal means of expressing them can be an effective way to foster engagement and ownership.

Technology affords other opportunities and challenges for the development and analysis of language. The symbols used in a spreadsheet may be related to, but are not the same as, the algebraic symbols used generally by mathematicians. Students will profit from experiences that require comparisons of standard mathematical expressions with those used with popular tools like spreadsheets or calculators.

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