MA 310 — Homework #6

Due Monday, March 2, in class

1. Solve "Binomial Coefficients" in the file "Problems" by doing the following: For non-negative integer n consider the expansion of

$$(x+y)^n = c_{n,0}x^n y^0 + c_{n,1}x^{n-1}y^1 + c_{n,2}x^{n-2}y^2 + \cdots + c_{n,n}x^0 y^n.$$

   We are going to figure out formulas for these coefficients.

   (a) Think carefully about the fact that $(x+y)(x+y)^{n-1} = (x+y)^n$. Then prove (without induction) that

$$c_{n,0} = c_{n,n} = 1, \text{ for all } n \geq 0,$$

   and

$$c_{n-1,k-1} + c_{n-1,k} = c_{n,k} \text{ for all } n \geq 1,\ 1 \leq k \leq n-1.$$

   (b) Now prove by induction on $n \geq 0$ that

$$c_{n,k} = \frac{n!}{k!(n-k)!},\ n \geq 0,\ 0 \leq k \leq n.$$

2. Solve "Choosing and Permuting" in the file "Problems."

3. Using the solution to the previous problem, solve "Choosing" in the file "Problems."

4. Read Section 3.1 on Symmetry in the text, and especially study Example 3.1.5. Now solve Problem 3.1.13. Include a neat and accurate sketch.

5. A triangle is inscribed in a given circle. Prove that if the triangle is not equilateral, then there is another triangle with larger area that can be inscribed in the same circle.