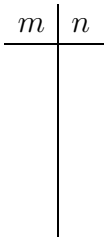
MA341 Homework #6
Chapter 3

Write your name on this sheet.

1. Find and justify a formula for the measure of the interior angle of a regular n-gon for arbitrary integer $n \geq 3$. (For an equilateral triangle, $n = 3$, it is 60 degrees. For a square, $n = 4$, it is 90 degrees.)

2. Three squares (4-gons) meet at each corner of a cube. So three interior angles, each measuring 90 degrees, are meeting at a corner, giving a total of 270 degrees. This is less than 360 degrees, which is why the corner is actually a corner of a convex polyhedron (and not flat). Find all ways in which m copies of the same regular n-gon, where $m \geq 3$, can fit together in a cluster at a common vertex with a total angle of less than 360 degrees.



3. Each of the above clusters can be extended to a regular polyhedron, having exactly the same cluster at each vertex. For example, the $(m, n) = (3, 4)$ cluster extends to make the cube. Use Polydron to construct one example of each of these polyhedra. These are called the *Platonic solids*.

4. Solve Problem 29 on page 127. It is helpful to use rubber bands.

5. Refer to Problem 40 on pages 130–131 and make a model of the stella octangula (two tetrahedra nested in a common cube) with Polydron. You don't need to construct the cube—just the two intersecting tetrahedra.

6. Refer to Problem 33 on page 128 and make a model of a truncated octahedron with Polydron.

7. Imagine what a truncated icosahedron must look like and make a model of it with Polydron or Zometool.

8. Look at the diagram in Problem 30, page 127, that shows a tetrahedron nested inside a cube (but don't do Problem 30 yet). Now use Zometool to construct a cube nested inside a dodecahedron.

9. A cube with side length equal to 2 is centered at the origin of an xyz coordinate system. Make a good sketch of the cube and label the eight vertices $P_1, \ldots, P_8$. Determine the (x, y, z)-coordinates of each of vertex.

10. Sketch the diagram for Problem 30 below and use it to determine coordinates for the vertices of a regular tetrahedron.

11. Sketch the diagram for Problem 32 below and use it to determine coordinates for the vertices of a regular octahedron.

Homework #6 Addendum
Solutions to Write Up by Monday, October 29

12. Write up the solution to 1. If you use the fact that the interior angles sum to $180(n-2)$ degrees, prove this also.

13. Give the short list of answers to 2 (there should be five of them). No explanation is required. Label each of these with the name of the corresponding Platonic solid.

14. To show your answers to 4, construct a cube out of posterboard or file folders (use some care in this). Carefully draw on the surface some lines showing how to get the following cross-sections, and label them (perhaps color code them). You may work in groups on this and submit one cube for each group—just be certain I know who contributed to each cube.

    (a) equilateral triangle
    (b) isosceles (but not equilateral) triangle
    (c) scalene triangle
    (d) rectangle
    (e) isosceles trapezoid (not a rectangle)
    (f) nonisosceles trapezoid
    (g) pentagon (but not necessarily regular)
    (h) regular hexagon

15. Make a careful sketch of the stella octangula from 5.

16. Verbally describe the truncated octahedron of problem 6—What types of faces does it have? How many of each type? Why? What combination of faces meets at each vertex? Why?

17. Verbally describe the truncated icosahedron of problem 7—What types of faces does it have? How many of each type? Why? What combination of faces meets at each vertex? Why?

18. Complete problem 9.

19. Complete problem 10.

20. Complete problem 11.