MA 341 Homework #8
Due Wednesday, December 5, in Class

Begin preparing for Exam #4 (the final exam, covering material on area, volume, and surface area), which will take place on on Wednesday, December 12, 10:30 am – 12:30 pm, in our regular room.

1. Derive the area formula for a general trapezoid with bases b_1 and b_2, and height h. Be sure your argument works for all trapezoids, not just isosceles trapezoids, and not just those for which the two sides both "point outward." [Key concept: Areas by dissection.]

2. Derive the surface area formula for a circular cylinder of radius r and height h. [Key concepts: Relating three-dimensional objects to two-dimensional objects; nets.]

3. Prove that if ΔCAB is a triangle with coordinates $C = (0,0)$, $A = (x_1, y_1)$, $B = (x_2, y_2)$, then its area is
$$\frac{1}{2}|x_1y_2 - x_2y_1|.$$
Suggestion: Use the triangle area formula $\frac{1}{2}ab\sin C$ and the cosine formula $\cos C = \frac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}}$ [Key concept: Relationship of area to determinants.]

4. C_1 and C_2 are concentric circles, with C_2 being the smaller circle. There is a chord of C_1 that is tangent to C_2, and the length of this chord is 10 units. What is the area of the region inside C_1 but outside C_2? [Key concept: Recognizing when provided information is sufficient.]

5. Derive the formula for the surface area of a circular cone of radius r and height h. Don't forget to include the area of the base. [Key concepts: Relating three-dimensional objects to two-dimensional objects; nets; approximation.]

6. Use calculus to determine the volume of a sphere of radius r. [Key concept: Volumes by slicing.]

7. (a) Use Cavalieri's principle to prove that the following volumes are equal: The volume of an upper hemisphere of radius r, and the volume of a cylinder of radius r and height r from which an inverted (upside down) cone of radius r and height r has been removed.

   (b) Use this to determine the formula for the volume of a sphere.

   [Key concept: Cavalieri's principle.]

8. (a) Motivate the formula $A = \frac{1}{2}Cr$ for a circle by slicing the circle into many thin pie-shaped pieces, where A is its area and C is its circumference.

   (b) Now think of something analogous for a sphere to motivate the formula $V = \frac{1}{3}Sr$, where V is its volume and S is its surface area.

   [Key concepts: Relation between area, circumference, volume, and surface area; analogies between two-dimensional and three-dimensional figures.]

9. (a) Take the derivative of the formula for the area of a circle of radius r. What formula do you get? Why does this happen?

   (b) Take the derivative of the formula for the volume of a sphere of radius r. What formula do you get? Why does this happen?

   [Key concept: Geometric understanding of derivative.]

10. Consider a solid figure X made by gluing together a number of unit cubes (cubes of side length 1 unit). Let Y be a figure similar to X, with the scaling factor 3 from X to Y. Use unit cubes to explain why it makes sense that the surface area of Y is 9 times the surface area of X, and the volume of Y is 27 times the volume of X. [Key concept: Effect of scaling on area, surface area, and volume.]

11. Use calculus to determine the volume of a pyramid whose base has area B and whose height is h. [Key concepts: Volumes by slicing; effect of scaling on area.]

12. You are given a square based pyramid from which the top (which was centered above the base) has been sliced off parallel to the base and removed, resulting in a flat square upper face. The base square has side length a, the top square has side length b, and the height between the bases is h. Find the volume of this object. [Key concept: Volumes by dissection.]

13. Find the radius and height of a cylinder with volume 100 cubic units that has the smallest surface area. [Key concept: Optimizing a function subject to a constraint.]

14. Use areas of particular figures made out of squares to prove that $1+3+5+\cdots+(2n-1) = n^2$ for all positive integers n. [Key concept: Making geometric models for algebraic statements.]

15. Extra Credit: Do some research (e.g., on the internet) to find a way to dissect an equilateral triangle into four pieces that can be reassembled into a perfect square. Construct these pieces carefully and accurately with physical material such as cardboard or wood.