Toves and Borogoves

1. Contact has been established with an alien race (the Carrollians) and they convey the following information to you about a mathematical structure of interest to them.

   A. There is a finite number of toves.

   B. There is a finite number of borogoves.

   C. Given any borogove there are exactly two different toves that gimble with it.

   Prove that the number of toves that gimble with an odd number of borogoves is even.

2. What is the meaning of toves and borogoves? What does gimble mean? What representations, if any, did you create while working on this problem?

3. **Handshaking.** At a recent conference, various pairs of people shook hands. Prove that the number of people who shook hands an odd number of times is even.

4. **Graphs.** A *graph* $G = (V, E)$ consists of a finite set V of *vertices* and a finite set E of *edges*. Assume there are no loops, so each edge joins two distinct vertices. The *degree* of a vertex is the number of edges joined to it. Prove that the number of vertices of odd degree is even.

5. **Lines.** Consider a finite collection $\mathcal{L}$ of lines in the plane with the property that no three pass through a common intersection point. Let $\mathcal{P}$ be any subset of the collection of all the various intersection points of these lines. Prove that the number of lines of $\mathcal{L}$ containing an odd number of points in $\mathcal{P}$ is even.

6. **Polyhedra.** Join together a finite collection of convex polygons edge to edge to enclose a region of space, with two polygons meeting at each edge. Prove that the number of polygons with an odd number of sides is even.