1 Counting Warm-Ups

1.1 Some Definitions

The *Cartesian product* of two sets X and Y is the set

$$X \times Y = \{(x, y) : x \in X, y \in Y\}.$$

That is, it is the set of all ordered pairs where the first coordinate comes from X and the second coordinate comes from Y.

A *relation* between two sets X and Y is *any* subset of $X \times Y$. That is, it is any subset of ordered pairs, where the first coordinates are drawn from X and the second coordinates are drawn from Y. The *domain* of a relation S is the set of first coordinates in the relation; i.e.,

$$\text{Domain}(S) = \{x \in X : (x, y) \in S \text{ for some } y \in Y\}.$$

The *range* of a relation S is the set of second coordinates in the relation; i.e.,

$$\text{Range}(S) = \{y \in Y : (x, y) \in S \text{ for some } x \in X\}.$$

Here are my high school teacher's definitions: A *ficklepicker* of a relation is a first coordinate that appears in more than one ordered pair of the relation. A *function* is a relation with no ficklepickers.

If f is a function, then we write, for example, $y_1 = f(x_1)$ to mean that the ordered pair (x_1, y_1) is in the function. When we write $f : X \longrightarrow Y$ is a function, we mean that the domain of f is the entire set X, and the range of f is contained in (but not necessarily equal to) Y.

A function $f : X \longrightarrow Y$ is *one-to-one* or an *injection* if no element of the range is paired with more than one element of X. A function $f : X \longrightarrow Y$ is *onto* or a *surjection* if its range is all of Y. A function $f : X \longrightarrow Y$ is *one-to-one and onto* or a *bijection* if it is both one-to-one and onto.

1.2 Some Counting Questions

When we write $|X| = m$, we mean that X is a finite set that contains exactly m elements; i.e., the cardinality of X is m.

1. If $|X| = m$ and $|Y| = n$, how many relations between X and Y are there?
2. If $|X| = m$ and $|Y| = n$, how many functions $f : X \longrightarrow Y$ are there?
3. Suppose $|X| = m$, $|Y| = n$. Try to fill in the following table by giving a formula for the number of functions $f : X \longrightarrow Y$ in each case:

	Injections	Surjections	Bijections
$m < n$			
$m = n$			
$m > n$			