MA415 Final Exam
Due NO LATER THAN Friday, December 17, 3:00 pm
PLEASE GIVE YOUR EXAM TO SARAH LEE IN ROOM 413 POT

This is a take-home exam. You may consult your text and your notes, and may ask me questions, but otherwise you may not use any other assistance, human or non-human.

1. Page 143, #5.

2. Page 144, #11.

3. Page 155, #5.

4. Page 162, #6.

5. Consider the following two player game. Initially there are two piles of chips, one with m chips, and the other with n chips, which we can represent by the ordered pair (m, n). Players alternate making moves. To make a move, a player may take either a positive number of chips from one pile, or else exactly one chip from each of the piles. The winner is the player to remove the very last chip, leaving the position $(0, 0)$, which is therefore a "good" position.

   (a) Determine which positions (i, j), $0 \leq i \leq 10$, $0 \leq j \leq 10$, are the "good" positions.

   (b) Determine and prove in general which positions (i, j) are good positions, $i \geq 0$, $j \geq 0$.