

MA/CS415 Homework #4
Due Wednesday, September 22

1. Read and study pages 30–51.
2. Page 25, #2, 3, 4.
3. Let G be a simple graph. Assume $v_1, v_2, \dots, v_{k-1}, v_k$ is a path of shortest length from v_1 to v_k . Prove that v_1, v_2, \dots, v_{k-1} is a path of shortest length from v_1 to v_{k-1} . While solving this problem, be careful to remember that a walk may not necessarily be a path.
4. Let G be a simple graph. Suppose every edge e of G is assigned a nonnegative cost $c(e)$. Define the cost of any walk to be the sum of the costs of its edges.
 - (a) Assume that $v_1, v_2, \dots, v_{k-1}, v_k$ is a walk of minimum cost from v_1 to v_k . Prove that there exists a path from v_1 to v_k having the same cost. Hint: Think about removing closed walks.
 - (b) Assume that $v_1, v_2, \dots, v_{k-1}, v_k$ is a path of minimum cost from v_1 to v_k . Prove that v_1, v_2, \dots, v_{k-1} is a path of minimum cost from v_1 to v_{k-1} . Again, while solving this problem, be careful to remember that a walk may not necessarily be a path.