Geometry #3

Before Tuesday, September 4

Read Dunham, Chapter 2, and read "Notes on Geometry" Sections 5.1–5.4. available from the course website http://www.ms.uky.edu/~lee/ma501fa07/ma501fa07.html. Go to the Forum "Axiomatics" and make at least one substantive contribution by 11 pm, Tuesday, September 4, and at least one substantive response to others' postings before class on Thursday, September 6. Respond to my musings on my "levels" in Section 5.3 by writing about the following: On which level do you place yourself? On what level should/can students be in various points in the K–16 timeline? What does this have to do with the van Hiele levels?

Before Thursday, September 6

Read Dunham, Chapter 2, and read "Notes on Geometry" Sections 5,4–5.4. available from the course website http://www.ms.uky.edu/~lee/ma501fa07/ma501fa07.html. Familiarize yourself with the website aleph0.clarku.edu/~djoyce/java/elements/elements.html. As you read, think about the following questions for discussion.

1. For axiom systems, what is the meaning of *consistent*, *independent*, and *complete*? What are *models* and how can they help determine whether axiomatic systems are consistent, independent, and/or complete?

2. Lewis Carroll, the author of *Alice's Adventures in Wonderland* and *Through the Looking-Glass* also wrote a book called *Euclid and His Modern Rivals*. What is this book about, and why did Lewis Carroll write it?

3. Why mention I Kings 7:23 on page 30 of Dunham?

4. Study some of the statements of Euclid's *Elements* on the website http://aleph0.clarku.edu/~djoyce/java/elements/elements.html, thinking about the wording and meaning, and studying the diagram. Does having a "dynamic" (draggable) diagram help in your understanding?

5. Look at some of the proofs of the propositions not covered in Dunham.

6. Try to carry out some of Euclid's constructions with Geogebra or other dynamic geometry software. Then try dragging around some of the points.

7. Try to find a website with which you can illustrate some of the statements of hyperbolic geometry—for example, one in which you can sketch a triangle and measure its angles.

8. Find a list of Hilbert's axioms for geometry. Find a list of the SMSG axioms for geometry (these are the ones my teacher used in my 10th grade geometry course). Find a list of axioms from some other geometry text. Compare these various lists with with those of Euclid.

9. Look at the material related to the Pythagorean Theorem on the National Library of Virtual Manipulatives, http://nlvm.usu.edu/en/nav/vlibrary.html.

10. Take a look at the website http://www.cut-the-knot.org/pythagoras/index.shtml.

11. Study Euclid's proposition VI.31, and relate this to your past homework.

12. What is the connection of the Pythagorean Theorem to the distance formula? Is this a standard connection typically made in High School?

13. Which basic trigonometric identity is actually a statement of the Pythagorean Theorem?

14. There is a song in the Gilbert and Sullivan comic opera *The Pirates of Penzance* mentioning the Pythagorean Theorem. Find these lyrics.

15. When the Scarecrow discovered he had a brain at the end of the *Wizard of Oz*, he quoted the Pythagorean Theorem. Or did he? Yes, even this can be found on the web!

16. The compasses commonly used in school hold a fixed opening, allowing lengths to be transferred from place to place. A Euclidean compass did not do that; as soon as it was picked up, the size of the span was lost. However, Propositions 2 and 3 of Book I prove that lengths can be transferred, thereby legitimizing modern compasses. Study these propositions to see how Euclid does this.

17. Find a list of axioms for:

    (a) Set theory (Zermelo-Fraenkel)

    (b) The natural numbers (Peano)

    (c) The real numbers (and why don't the rational numbers satisfy all of them?)

    (d) Groups

    (e) Vector spaces

Name some college level mathematics courses that typically begin with a set of axioms for some collection of objects.

Thursday, September 6, 7–9 pm

Attend the Adobe Connect session to discuss the readings, discussion questions, forum, and comments and questions on the assigned homework due on Sunday.

Before Sunday, September 9, 11 pm

Homework problems due Sunday, September 9, 11 pm, uploaded to the Moodle site as a single file less than 2 MB, or else emailed to the address lee@ms.uky.edu. Please use Word or pdf files only.

1. *Notes on Geometry*, Exercise 5.2.1.
2. *Notes on Geometry*, Exercise 5.4.1.
3. *Notes on Geometry*, Exercise 5.4.3. (I don't remember the answer to this one!)
4. *Notes on Geometry*, Exercise 5.4.4.
5. The ratio of the circumference of a circle to its diameter is not π in non-Euclidean geometry. In fact, it is not even a constant. Illustrate this by considering a sphere of radius 1, marked with lines of latitude and longitude as on a standard globe.
   (a) Look at the circle A of 45 degrees north latitude (45 degrees up from the equator). *Regard the north pole N as its center, and a great circle arc from N to the circumference of the circle A as its radius.* Calculate the ratio of the circumference to the diameter, giving an exact answer, and then also evaluating it to several decimal places.
   (b) Repeat with the circle B that is 30 degrees north latitude.
   (c) Repeat with the circle C that is 0 degrees north latitude (i.e., the equator).
   (d) Try to develop a formula for the ratio of the circumference to the diameter of a circle of x degrees N latitude. How does this number compare to π?
6. Extra Credit. *Notes on Geometry*, Exercise 5.4.7.