MA 501 Homework #4

Due Tuesday, February 4, in class

In this set of problems the aim is to understand what the net effect is of performing two isometries in a row, and ultimately what kinds of symmetries border (strip, or frieze) patterns can have. GeoGebra is helpful for experimention, but not required.

Recall first that we know about four different kinds of isometries for the plane:

- Translations. Move the entire plane by a certain distance in a certain direction. (This motion can be represented by a given vector.) Note that the identity isometry is sometimes regarded as special case of a translation (moving the plane by a distance zero).
- Rotations. Rotate the entire plane by a certain amount (the angle of rotation) about a certain point (the center of rotation). Note that the identity isometry is sometimes regarded as special case of a rotation (by zero degrees).
- Reflections. Reflect the entire plane with respect to a certain line (the line of reflection).
- Glide Reflections. First reflect the entire plane with respect to a certain line (the line of reflection) and then translate the plane parallel to that line by a certain amount. Note that an ordinary reflection is sometimes regarded as a special case of a glide reflection (having no translation).

Let's focus on a subset of the isometries—those that can map a strip in the plane to itself. The strip will be a the set of points centered on the x-axis with a width of two units—all points with coordinates (x, y), where $-1 \leq y \leq 1$.

Notice that there are the following isometries that place the strip back onto itself:

- The identity isometry, in which each point remains at its current location.
- Horizontal translation to the right by a certain distance a, where a is real number. Let's denote this by T_a. (If a is negative the translation is actually to the left.)
- Rotation by 180 degrees about the point $(b, 0)$. Let's denote this by R_b.
- Vertical revlection across the line $x = c$, where c is a real number. Let's denote this V_c.
- Horizontal reflection across the x-axis. Let's denote this by H.

- Glide reflection across the x-axis, followed by translation to the right by the amount d, where d is a real number. (So if d is negative, the translation is actually to the left.) Let's denote this by G_d.

1. Do Exercises 8.20 and 8.21 in the document `http://www.ms.uky.edu/~lee/ma241/ma241notesb.pdf`. Except for 8.2.1 parts (8) and (9), no explanation is required.

2. For each of the following pairs of isometries on a strip, list the possible net effects of performing these two isometries one after the other. You do not need to provide an explanation.

   (a) Translation, translation. Answer: Either a translation or the identity.

   (b) Translation, rotation. Answer: Rotation.

   (c) Translation, vertical reflection.

   (d) Translation, horizontal reflection.

   (e) Translation, glide reflection.

   (f) Rotation, rotation (not necessarily about the same points).

   (g) Rotation, vertical reflection (the vertical reflection is not necesssarily through the center of rotation).

   (h) Rotation, horizontal reflection.

   (i) Rotation, glide reflection.

   (j) Vertical reflection, vertical reflection (not necessarily across the same lines).

   (k) Vertical reflection, horizontal reflection.

   (l) Vertical reflection, glide reflection.

   (m) Horizontal reflection, horizontal reflection.

   (n) Horizontal reflection, glide reflection.

   (o) Glide reflection, glide reflection (not necessarily by the same amount).

3. Read "The Symmetries of Culture" `http://vismath6.tripod.com/crowe1`.