MA515 Homework #1
Due Monday, August 30

1. A *matching* in a graph is a subset of edges of the graph such that no two selected edges share a common endpoint. If we associate the edges $e \in E(G)$ of the graph with variables x_e, consider the characteristic vectors of matchings, and try to find inequalities satisfied by these vectors we readily see that the following must hold:

$$\sum_{e \in \delta_G(v)} x_e \leq 1 \text{ for all } v \in V(G),$$

$$x_e \geq 0 \text{ for all } e \in E(G).$$

(Please refer to the Appendices for the notation.)

Consider the graph G that is a cycle of three vertices and three edges—a "triangle". Draw the polyhedron associated with the above inequalities. Do the extreme points of this polyhedron precisely correspond to the matchings in the graph, or not? If not, what additional inequality or inequalities are needed to create a polyhedron whose extreme points correspond precisely to the matchings in the graph?

2. Read the Introduction of the text.