MA515 Homework #5
Due Monday, October 16

1. Let us assume we have an LP of the form

$$\begin{aligned} \max z &= c^T x \\ \text{s.t. } Ax &= b \\ x &\geq O \end{aligned}$$

   (a) Let $\overline{x}$ be a feasible point. Prove that $\overline{x}$ is a basic feasible solution if and only if it is a vertex (using our earlier definition of vertex involving $N(\overline{x})$).

   (b) Assume that we have a basic feasible solution $\overline{x}$ associated with some basis B, and that we also have some basic direction $\overline{w}$ associated with B and nonbasic $s \in N$. For convenience, let us also assume that $\overline{x}_j > 0$ for each $j \in B$ and that $\overline{w}$ is not nonnegative. So when we consider the ray $\overline{x} + t\overline{w}$, $t \geq 0$, we will discover some leaving variable x_r, $r \in B$. Prove that $B' = (B \cup \{s\}) \setminus \{r\}$ is a basis; i.e., prove that the columns of $A_{B'}$ are linearly independent.

2. Exercise 8.3.

3. Exercise 8.8.

4. Exercise 10.2.