MA515 Homework #8
Due Wednesday, November 22

1. In a communications network, represented by a digraph, the probability that the edge from i to j is operative is p_{ij}. The probability that all the edges in any given dipath are operative is the product of the edge probabilities. How can one solve the problem of finding a most reliable dipath from one designated vertex to all of the others? Justify your answer.

2. Suppose a digraph is given with nonnegative capacities c_{ij} assigned to the edges. The capacity of a dipath is defined to be the minimum of the capacities of its edges. How can one solve the problem of finding maximum capacity dipaths between every pair of vertices? Justify your answer.

3. SKIP THIS PROBLEM. Exercise [Scheduling, continued], page 99.

4. Consider the graph G with three vertices a, b, c and three edges $e_1 = ab, e_2 = ac, e_3 = bc$. Consider the matroid M_i, $i = a, b, c$, each with ground set $E = \{e_1, e_2, e_3\}$, given by $\mathcal{I}(M_i) = \{S \subseteq E : \text{no more than one edge of } S \text{ is incident to vertex } i\}$. Prove that the intersection of the three matroid polytopes $\mathcal{P}_{\mathcal{I}(M_a)}$, $\mathcal{P}_{\mathcal{I}(M_b)}$, and $\mathcal{P}_{\mathcal{I}(M_c)}$ contains at least one noninteger vertex, and hence

$$\mathcal{P}_{\mathcal{I}(M_a)} \cap \mathcal{P}_{\mathcal{I}(M_b)} \cap \mathcal{P}_{\mathcal{I}(M_c)} \neq \mathcal{P}_{\mathcal{I}(M_a) \cap \mathcal{I}(M_b) \cap \mathcal{I}(M_c)}.$$

5. Let G be a graph. Assume that every vertex is incident to at least one edge (we say that G has no *isolated vertices*). A subset X of edges is called a *cover (of vertices by edges)* if every vertex of G is incident to at least one edge in X. A subset Y vertices is called a *vertex packing* if no two vertices in Y are joined by an edge of G. Construct a proof, in a manner similar to the proof in class of König's Theorem, that for a bipartite graph with no isolated vertices, the size of a minimum cardinality cover is equal to the size of a maximum cardinality vertex packing.

6. On your own: Be sure you practice carrying out the various minimum weight dipath algorithms on concrete examples.