

MA/STA515 Homework #4

Due Monday, September 27

1. Problem 2D.
2. Problem 3A.
3. Let G be a simple graph with vertex set $V = \{1, \dots, n\}$. Let M be the $n \times n$ matrix given by

$$M_{ij} = \begin{cases} 0 & \text{if } i = j, \\ 1 & \text{if } i \neq j \text{ and } \{i, j\} \in E(G), \\ +\infty & \text{if } i \neq j \text{ and } \{i, j\} \notin E(G). \end{cases}$$

For any compatibly-shaped matrices A and B Define “weird” matrix multiplication by $AB = C$, where the product is given by

$$C_{ij} = \min_k \{A_{ik} + B_{kj}\}.$$

Find and prove a combinatorial interpretation of the entries of M^m for all $m \geq 1$, when the power of M is computed by the rules of weird matrix multiplication.

4. On your own: Doublecheck your understanding of all of the details of Proof 3 on pages 16–17.