Homework #3
Due Friday, February 8

1. Exercise 4.2.
2. Exercise 4.4.
3. Exercise 4.5.
4. Exercise 4.6. (You may use the result of Exercise 4.7.)
5. Exercise 4.10, which I have rewritten slightly here: Finish the proof of Gram's Theorem (Section 2.4) by showing that

$$\sum_{F:0\leq \dim F\leq d} (-1)^{\dim F} a_F(x) = 0$$

if $x \notin P$. Suggestion: Suppose P is a d-polytope in $\mathbf{R}^d$. Each facet F_i has a unique supporting hyperplane H_i. Let H_i^+ be the closed halfspace associated with H_i containing P, and H_i^- be the opposite closed halfspace. Let F be any proper face of P. Define x to be *beyond* F (or F to be *visible* from x) if and only if there is at least one i such that $F \subset F_i$ and $x \in H_i^- \setminus H_i$. Note that $a_F(x) = 0$ if and only if F is visible from x. Now prove that the set of faces visible from x is shellable. Apply Euler's relation (Exercise 4.6).