Assignment #11
Due Wednesday, April 20

REMEMBER TO COME TO THE HARDYMON BUILDING AT 3:00 ON WEDNES-
DAY, APRIL 20

1. In class we discussed winning and losing positions of games. The winning positions are
   the ones you want to achieve at the conclusion of your move. A position is winning
   if every position reachable from it in one move is losing. A position is losing if there
   is at least one winning position reachable from it. Here's another way to look at
   it: If you achieve any winning position, then no matter what move your opponent
   next makes, it is possible for you to make a responding move that brings you back to
   another winning position. Consider, for example, the game in which you start from
   0, alternately adding positive whole numbers from 1 to 6, with the goal of reaching
   100. The winning positions are those that are 2 greater than multiples of 7: 2, 9, 16,
   ..., 100. If you reach any of these winning positions, no matter what number k your
   opponent adds, you can add the number $7 - k$ to reach the next winning position.

   Determine and describe the set of winning positions of the following games:

   (a) There are two piles of candy. One contains 20 pieces, and other 21. Players take
       turns eating all the candy in one pile, and separating the remaining candy into
       two (not necessarily equal) non-empty piles. The player who cannot move loses.

   (b) Of two piles of stones, one contains 7 stones, and the other 5. Players alternate
       taking any number of stones from one of the piles, or an equal number from each
       pile. The player who cannot move loses.

   (c) The number 60 is written on a blackboard. Players take turns subtracting from the
       number on the blackboard any of its divisors, and replacing the original number
       with the result of this subtraction. The player who writes the number 1 wins.

2. If you haven't already, send me your POV-Ray 24-cell!! Use POV-Ray to produce an
   image of the 24-cell. Try to find some interesting directions of projection. You may
   also wish to include some of the triangular faces to help visualize this object. As usual,
   email me the source file.