Assignment #8

To write up and turn in:

Use the procedure we discussed in class to determine the cross-section of the cube having vertices $(\pm 1, \pm 1, \pm 1)$ with the plane having equation $x + y + z = 0$. Here are the steps:

1. First classify each vertex of the cube according to whether it is on the plane, or on one side or the other.

2. Using this information, determine which edges of the cube pass through the plane, and determine the coordinates of the points of intersection of these edges with the plane.

3. Make a sketch of the polygon that is the cross-section, labeling each vertex with its coordinates that you found in part (2.).

4. The vector $w = (1, 1, 1)$ is perpendicular to the plane (these are the coefficients of x, y and z in the equation of the plane), so we need two other vectors u and v each perpendicular to w and perpendicular to each other. Let's choose $u = (1, -1, 0)$. Use a cross-product to find v.

5. By dividing by their current respective lengths, rescale u and v so that they each have length one, getting vectors $\overline{u}$ and $\overline{v}$.

6. Find the two-dimensional coordinates of the projections of each of the vertices of the polygon in part (3.) by taking dot products with $\overline{u}$ and $\overline{v}$.

7. Verify that the cross-section is indeed a perfectly regular hexagon centered at the origin. (What is the distance of each of its vertices from the center? You may also want to refer up your notes and homework about sines, cosines, and coordinates of regular polygons.)