1 FIFTEEN

One player has five pennies, the other five dimes. Players alternately cover a number from 1 to 9. You win by covering three numbers somewhere whose sum is 15 (see Figure 1).

1 2 3 4 5 6 7 8 9

Figure 1: The Game of Fifteen

2 JAM

Players alternately select one of the nine roads (straight lines) and whoever manages to take all the roads through some town wins (see Figure 2). Note

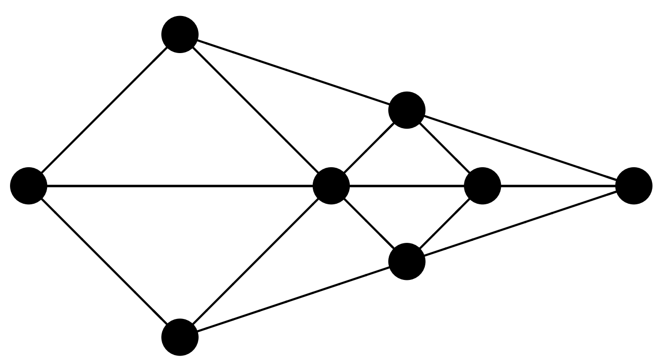

Figure 2: The Game of Jam

that most roads pass through at least one town. When you select a road, you select the *entire* road, and not just a single segment joining two adjacent towns.

3 SPIT NOT SO, FAT FOP, AS IF IN PAN!

Write the above nine words on nine separate cards and have two players alternately select cards, a player winning if he/she can collect all the cards which contain a given letter.

4 RELATIVES OF TICKTACKTOE

4.1 THREE-DIMENSIONAL TICKTACKTOE

This is familiar to most people. It is played on a $3 \times 3 \times 3$ board with the object of getting three in a row, or on a $4 \times 4 \times 4$ board with the object of getting four in a row.

4.2 WILD TICKTACKTOE

This is the same as Ticktacktoe, except that on your turn you may place either an X or an O—your choice—in an empty cell. Try this also with a $3 \times 3 \times 3$ board.

4.3 TOETACKTICK

This is the same as Ticktacktoe, except that the first player to get three in a row *loses.* Try this also with a $3 \times 3 \times 3$ board.

4.4 WILD TOETACKTICK

This is the same as Wild Ticktacktoe, except that the first player to get three in a row *loses.* Try this also with a $3 \times 3 \times 3$ board.

4.5 FOUR-DIMENSIONAL TICKTACKTOE

Four-dimensional Ticktacktoe can be played on an imaginary hypercube by sectioning it into two-dimensional squares. A $4 \times 4 \times 4 \times 4$ hypercube, for example, would be diagramed as shown in Figure 3. On this board a win of four in a row is achieved if four marks are in a straight line on any cube that

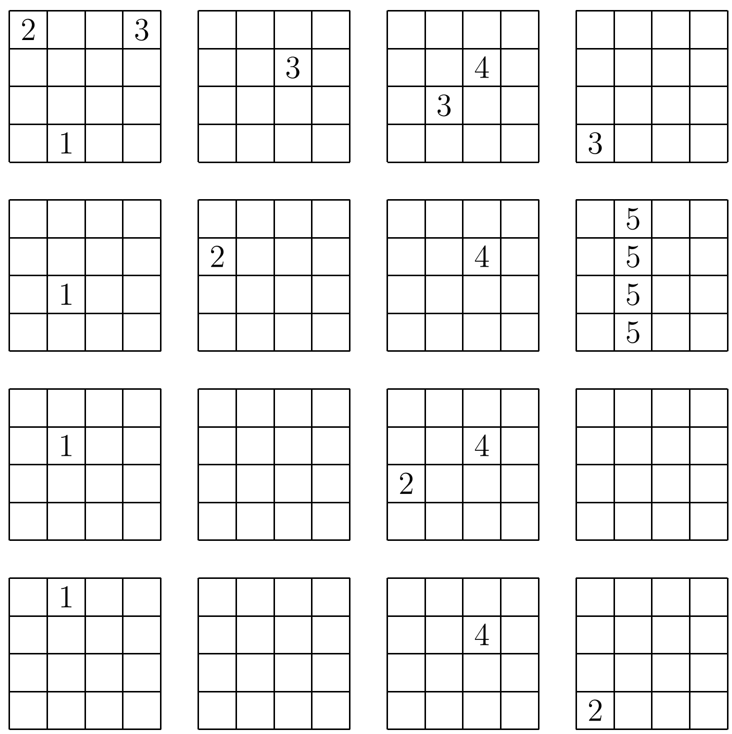

Figure 3: Four-Dimensional Ticktacktoe

can be formed by assembling four squares in serial order along any orthogonal or either of the two main diagonals. Figure 3 shows five examples of winning configurations. For example, if you occupy the four cells labeled 2, you win.

You can extend constructions of this type to play Ticktacktoe of any dimension!

5 STRING SNIPPING

We are given an arbitrary number of pieces of string of arbitrary lengths. Players take turns cutting a one-inch segment from any piece of string. The segment can be cut from the end or it can be cut, with two snips, from the interior. In the second case it will leave two pieces of string where there was one before. A one-inch piece can, of course, be taken without any snipping. The person who gets the last one-inch piece wins.

String lengths need not be rational. For example, you could begin with four pieces of string of lengths 1, π, the square root of 30, and the square root of 50.

The game can also be played with closed loops of string. For example, you could start with seven loops, each with a length greater than two inches.

6 DOMINO PLACING

There are two players, each with a large supply of dominoes. Players alternately move by placing one of their dominoes anywhere on a rectangular board. Each domino must be put down flat, within the border of the rectangle and without moving a previously placed piece. There are enough dominoes to cover the board completely when the pieces are packed side by side. The player who is unable to put down a domino loses.

7 CIGAR PLACING

There are two players, each with a large supply of cigars. Players alternately move by placing one of their cigars anywhere on a rectangular table. Each cigar must be placed within the border of the table and without moving a previously placed cigar. No cigar may be placed on top of another. The player who is unable to put down a cigar loses.

8 COIN STEALING

Any number of coins are arranged in a circle on the table, each coin touching two of its neighbors. Players alternately remove either one coin or two touching coins. The player who takes the last coin wins.

9 FOCUS

Focus is played with 36 counters, half of them one color and half another. Small poker chips of the interlocking variety make excellent pieces. They are placed initially on an eight-by-eight board from which three cells at each corner have been removed. Figure 4 shows how the pieces are arranged.

Either side may move first. A move consists of moving a "pile" (at the outset all piles are one chip high) as many spaces as there are pieces in the pile. The four possible moves of one white piece at the start of the game are shown in Figure 4. If the piece moves up, it lands on a vacant square. a move to the right puts it on top of another white piece, to the left or down puts it on top of a black piece. The last three moves form two-high piles. Such piles may be moved two spaces in any direction. Piles of three, four, and five pieces move three, four, and five spaces respectively. A pile is controlled by the player who owns the piece on top. In moving it does not matter whether the intervening cells are empty or occupied by piles controlled by either player. Passed-over pieces are not affected in any way. A move may end on a vacant cell or on another pile.

Piles may not contain more than five pieces. If a move produces a pile of more than five, all pieces in excess of five are taken from the bottom of

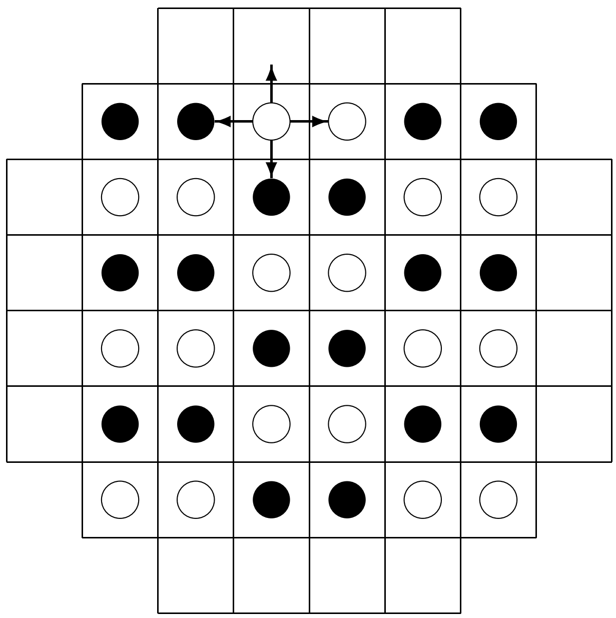

Figure 4: The Game of Focus

the stack. If they are enemy pieces, they are considered captured and are removed from the game. If they belong to the player making the move, they are placed aside as reserves. At any time during the game a player may, if he wishes, take one of his reserve pieces and place it on any cell of the board, empty or otherwise. It has the same effect as a moved piece: if it goes on a pile, the pile belongs to the player who placed it. Using a reserve piece substitutes for a move on the board.

A player may, if he wishes, make a move of fewer spaces than the number of pieces in the pile being moved. He does this by taking from the *top* of the pile as many pieces as the number of spaces he wishes to move. The rest of the pieces stay where they are. For example, a player may take the top three pieces of a five-high pile and move them three squares. The pile that *remains* after such a move belongs to the player who owns the piece on top.

When a player is unable to move (that is, controls no piles and has no reserves), the game is over and his opponent wins.

10 SLITHER

The game of Slither is played on a lattice. The five-by-six point lattice is a good size to try (see Figure 5). Opponents take turns marking a horizontal or vertical line segment joining two adjacent points. The segments must form a continuous path but may be added to either end of the preceding path. The player forced to close the path (i.e., the path cannot be extended without touching itself) loses. Figure 6 shows a typical position in which the next play must be a losing one.

The game can also be played so that the player who can cause the path to touch itself wins.

11 BLACK

Black is easily played as a pencil-and-paper game on a checkered field. The size of the field is optional; the standard eight-by-eight field seems ideal (see Figure 7, but it is simpler to explain the game on the smaller four-by-four.

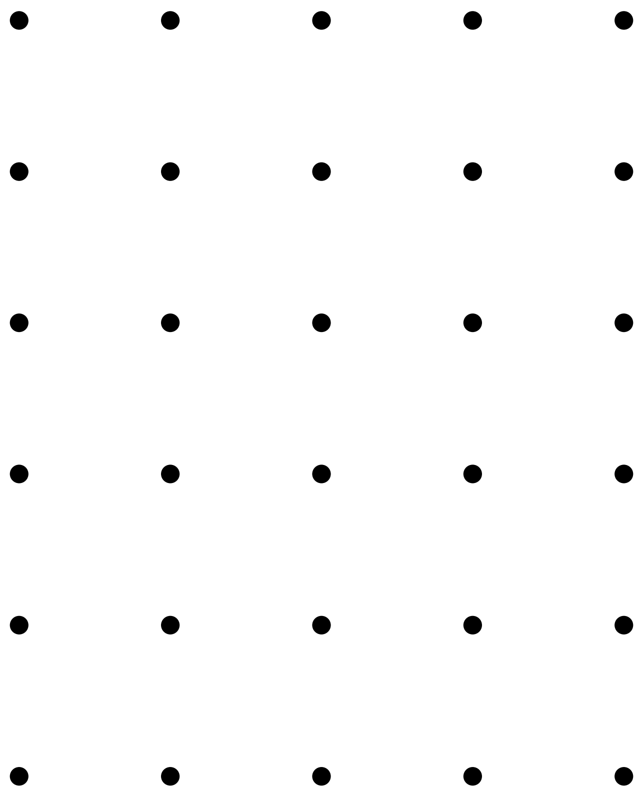

Figure 5: The Game of Slither

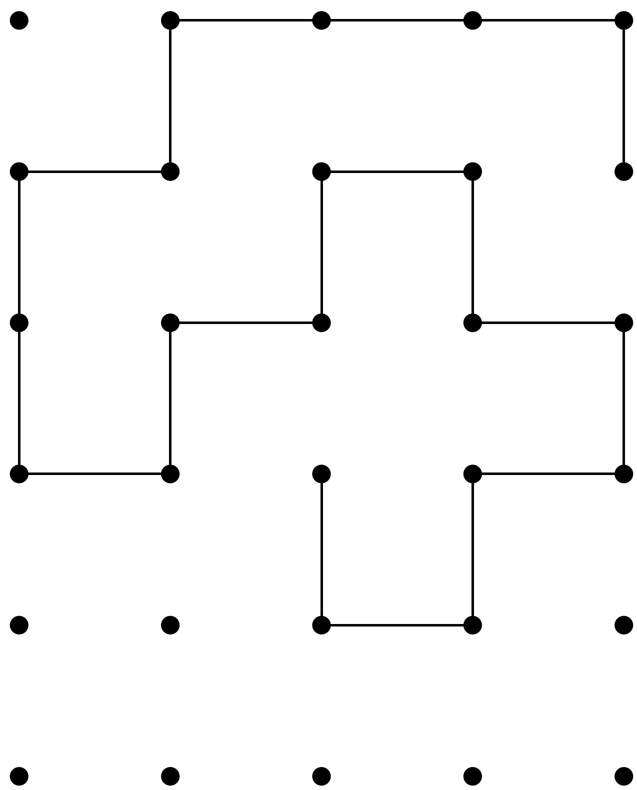

Figure 6: The End of a Slither Game

After the field is drawn the first player starts the game by making a cross in

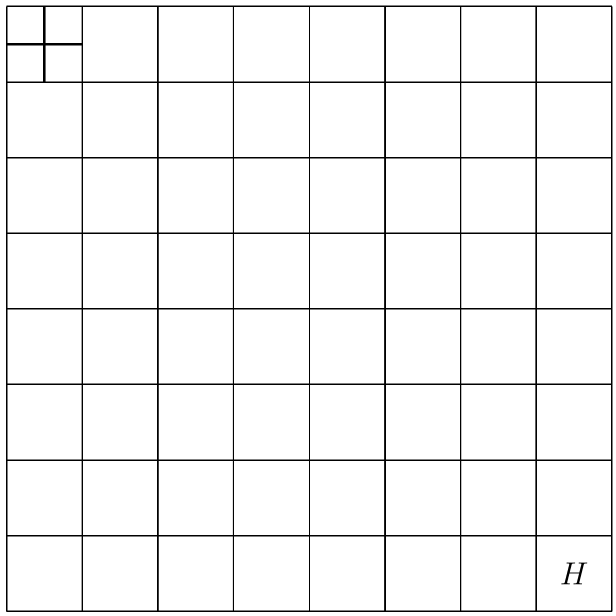

Figure 7: The Game of Black

the upper left corner cell as shown in Figure 8. The second player continues the path by making one of three permissible marks in a cell adjacent to the first cell marked. The three marks, shown at the bottom of the illustration, are each composed of two lines. One line represents one of the three ways in which the path can be joined to an open side of the square; the second is added to connect the remaining two sides.

The players alternate moves. Each move must extend the path into a neighboring cell. Each player tries to avoid running the path to a border of the field. If he is forced to carry the path to the border, he loses the game. He wins if he succeeds in extending the path into (not just to the border of) the lower right corner cell (marked by an H in Figure 7). Figure 8 shows successive moves of a typical short game. The first player wins by forcing his opponent to play in the upper right corner cell, where any mark will carry the path to the edge of the field. (Note that the cross extends the path only

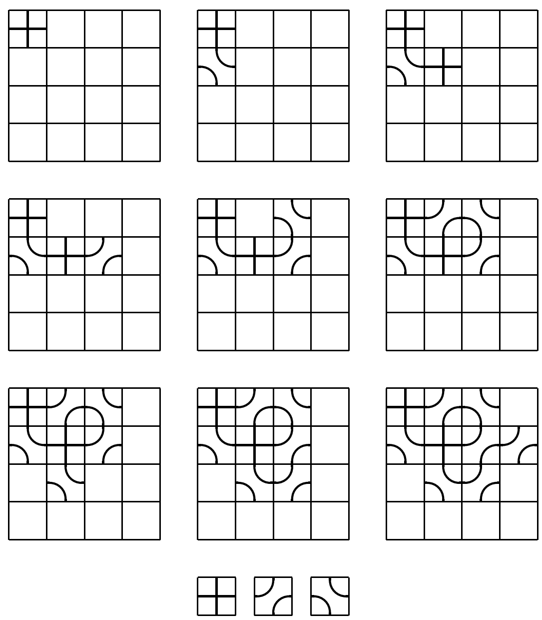

Figure 8: A Game of Black

along one its arms, although the other arm may become part of the path as the result of a later play.)

12 BRIDG-IT

A Bridg-it board is shown in Figure 9. If it is played on paper, one player

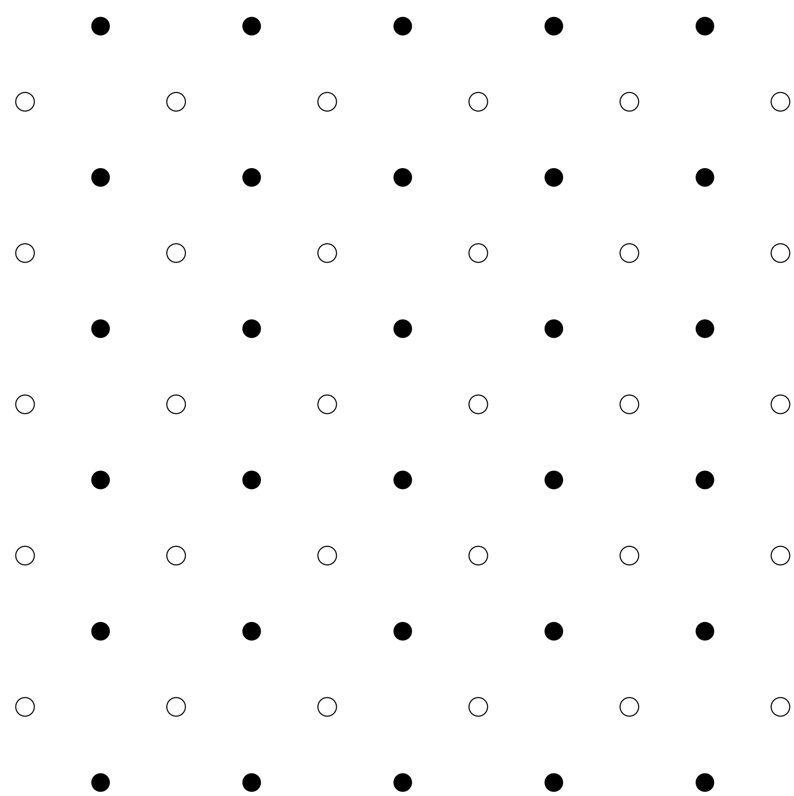

Figure 9: The Game of Bridg-it

uses a black pencil for drawing a straight line to connect any pair of adjacent black dots, horizontally or vertically but not diagonally. the other player uses a red pencil for similarly joining pairs of red dots. Players take turns drawing lines. No line can cross another. The winner is the first player to form a connected path joining the two opposite sides of the board that are his color. Figure 10 shows the result of a game in which red has won.

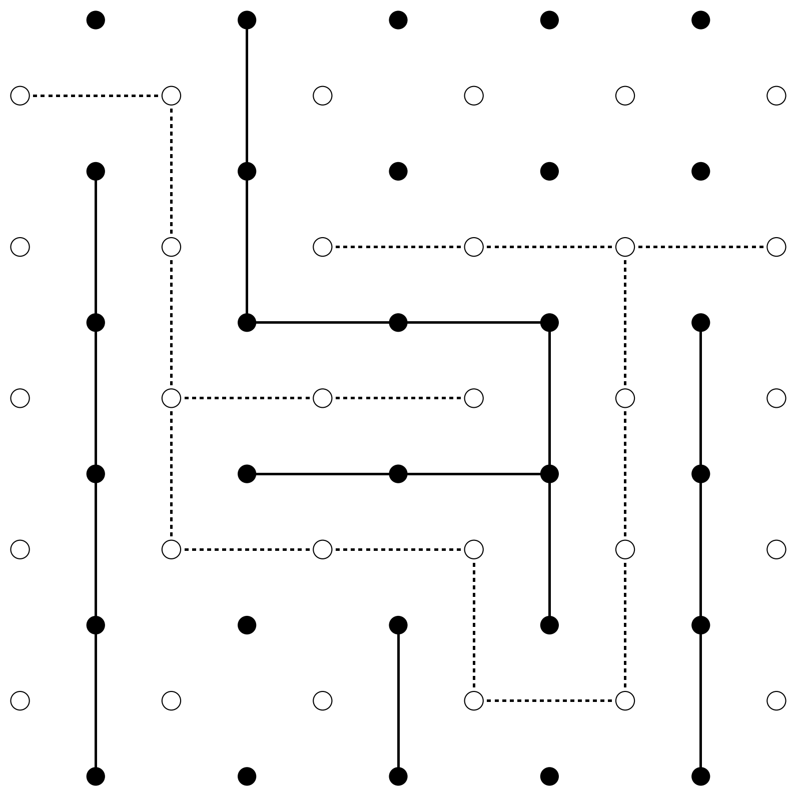

Figure 10: A Bridg-it Game in which Red has Won

13 HEXAPAWN

Hexapawn is played on a 3×3 board, with three chess pawns on each side as shown in Figure 11. Checkers or coins can be used instead of actual chess pieces. Only two types of move are allowed: (1) A pawn may advance straight forward one square to an empty square; (2) a pawn may capture an enemy pawn by moving one square diagonally, left or right, to a square occupied by the enemy. The captured piece is removed from the board. These are the same as pawn moves in chess, except that no double move, *en passant* capture or promotion of pawns is permitted.

The game is won in any of three ways:

1. By advancing a pawn to the third row.

2. By capturing all enemy pieces.

3. By achieving a position in which the enemy cannot move.

Players alternate moves, moving one piece at a time.

14 PENNY CHASE

Place a penny and a dime on the indicated spots of Figure 12.

Two players alternate turns, one moving the penny, the other the dime. Moves are made along a solid black line to an adjacent spot. The penny player always moves first. His object is to capture the dime by moving onto the spot occupied by the dime. To win he must do so before he makes his seventh move. If after six of his moves he has failed to catch the dime, he loses.

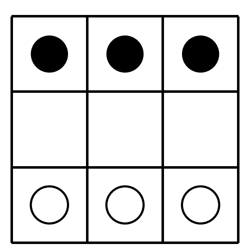

Figure 11: The Game of Hexapawn

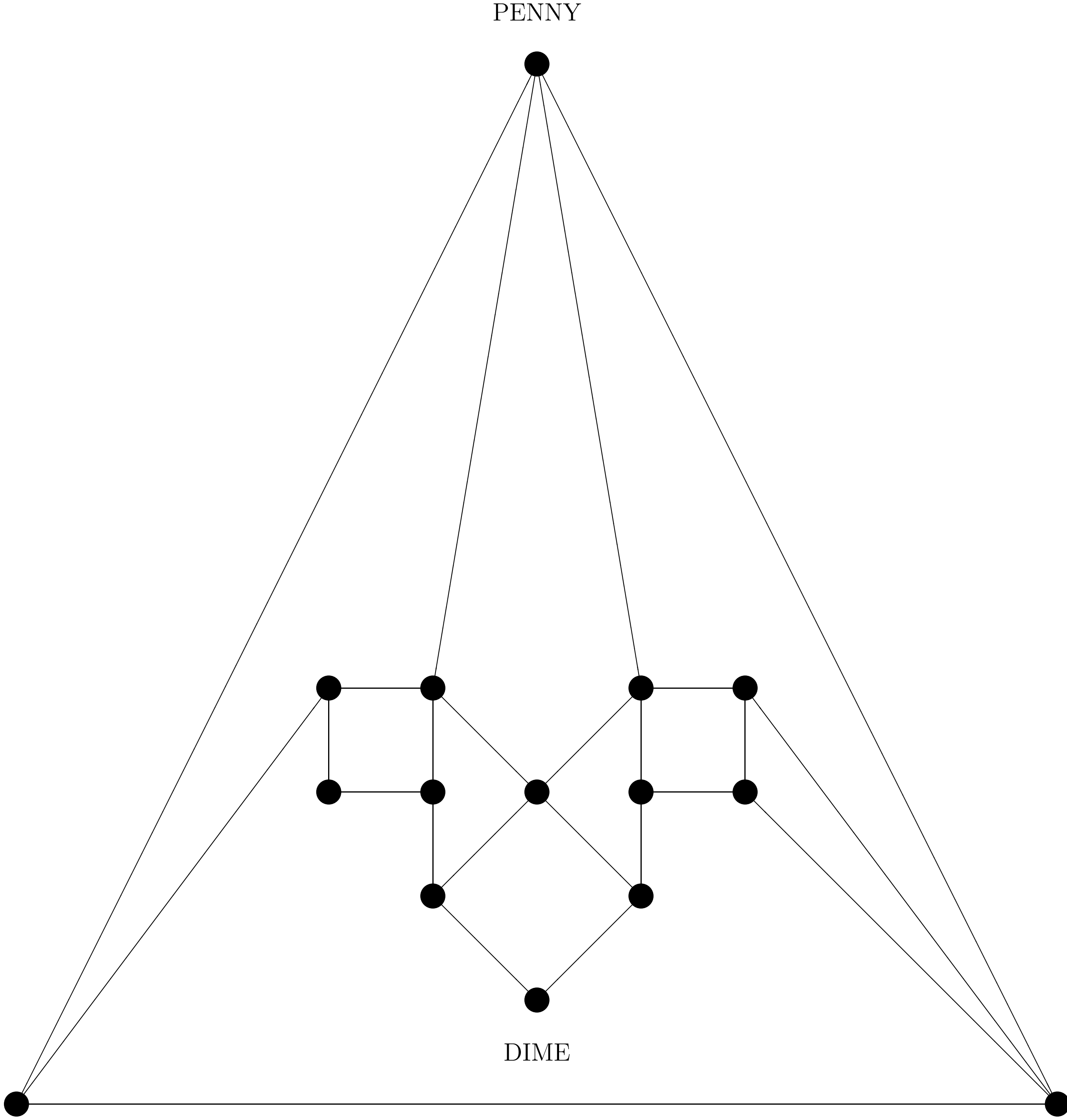

Figure 12: Penny Chase

15 FIBONACCI NIM

Fibonacci Nim begins with a pile of n counters. The two players take turns removing counters. The first player may not take the entire pile, but thereafter either player may remove all the remaining counters if these rules permit: at least one counter must be taken on each play, but a player may never take more than twice the number of counters his opponent took on his last play. Thus if one player takes three counters, the next player may not take more than six. The person who takes the last counter wins.

16 EUCLID

Two players agree upon a pair of unequal positive integers. When it is your turn, you may subtract any multiple of the smaller integer from the larger one (as long as no negative difference results). This leaves a new pair of integers, and the play continues, alternating between the two players. The first player to reduce one of the integers to zero is the winner.

17 DIE TURNING

An arbitrary number, usually larger than 20 to make the game interesting, is agreed upon. The first player rolls the die, scoring the number that is uppermost. The second player now gives the die a quarter turn in any direction, adding to the previous score the number he brings to the top. Players alternate in making quarter-turns, keeping a running total, until one of them wins by reaching the agreed-upon number or forcing his opponent to go above it.

18 SIM

Six points are placed on a sheet of paper to mark the vertices of a regular hexagon. There are 15 ways to draw straight lines connecting a pair of points, producing what is called the complete graph for six points. Two Sim players take turns drawing one of the 15 edges of the graph, one using a black pencil, the other using red. The first player to be forced to form a triangle of his own color (only triangles whose vertices are among the six starting points count) is the loser.

19 HEX

Hex is played on a diamond-shaped board made up of hexagons (see Figure 13). The number of hexagons may vary, but the board usually has 11 on each edge. Two opposite sides of the diamond are labeled “black”; the other two sides are “white.” The hexagons at the corners of the diamond belong to either side. One player has a supply of black pieces; the other, a supply of white pieces. The player alternately place one of their pieces on any one of the hexagons, provided the cell is not already occupied by another piece. The objective of “black” is to complete an unbroken chain of black pieces between the two sides labeled “black.” “White” tries to complete a similar chain of white pieces between the sides labeled “white.” See Figure 14 for an example of a path. There will almost certainly be pieces not on the winning path, and there is no obligation to construct the path in any particular order. All that is necessary to win is to have a path joining your two sides of the board somewhere among all the pieces you have played.

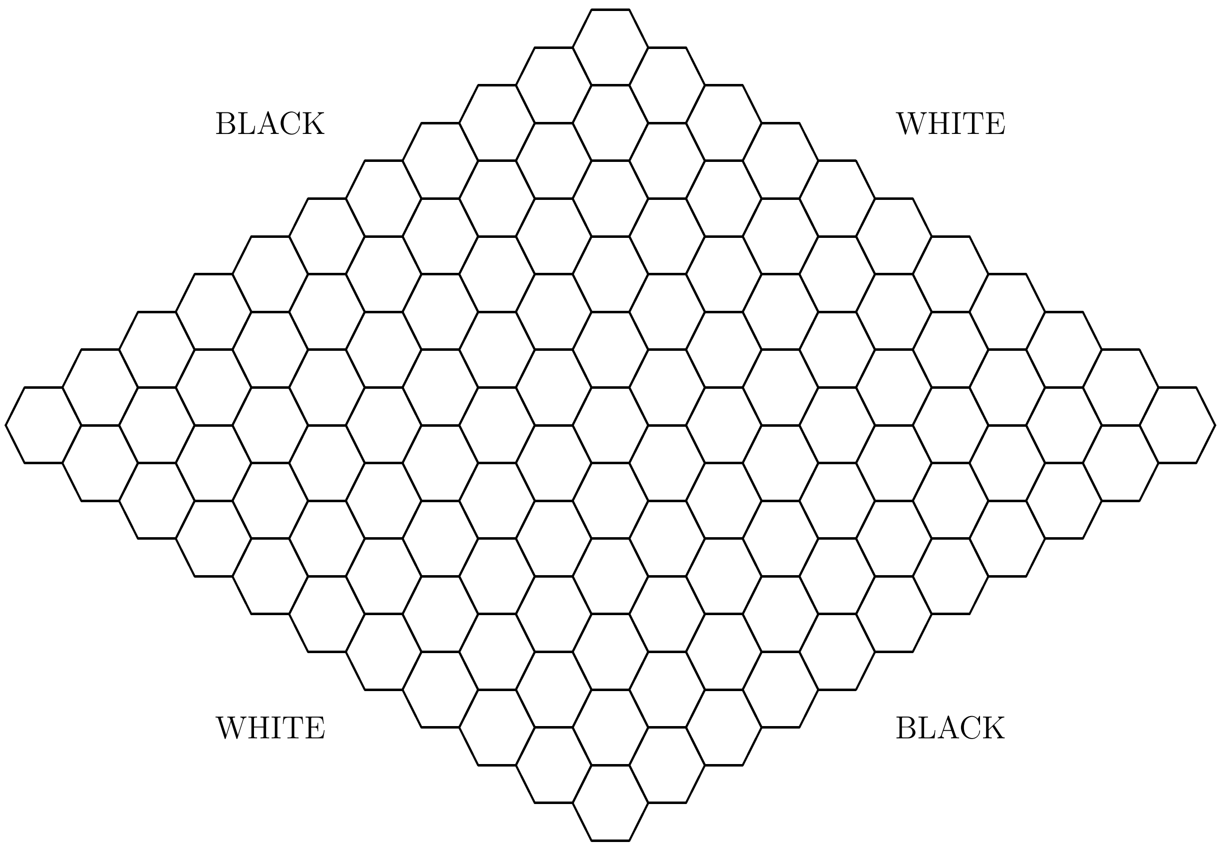

Figure 13: The Game of Hex

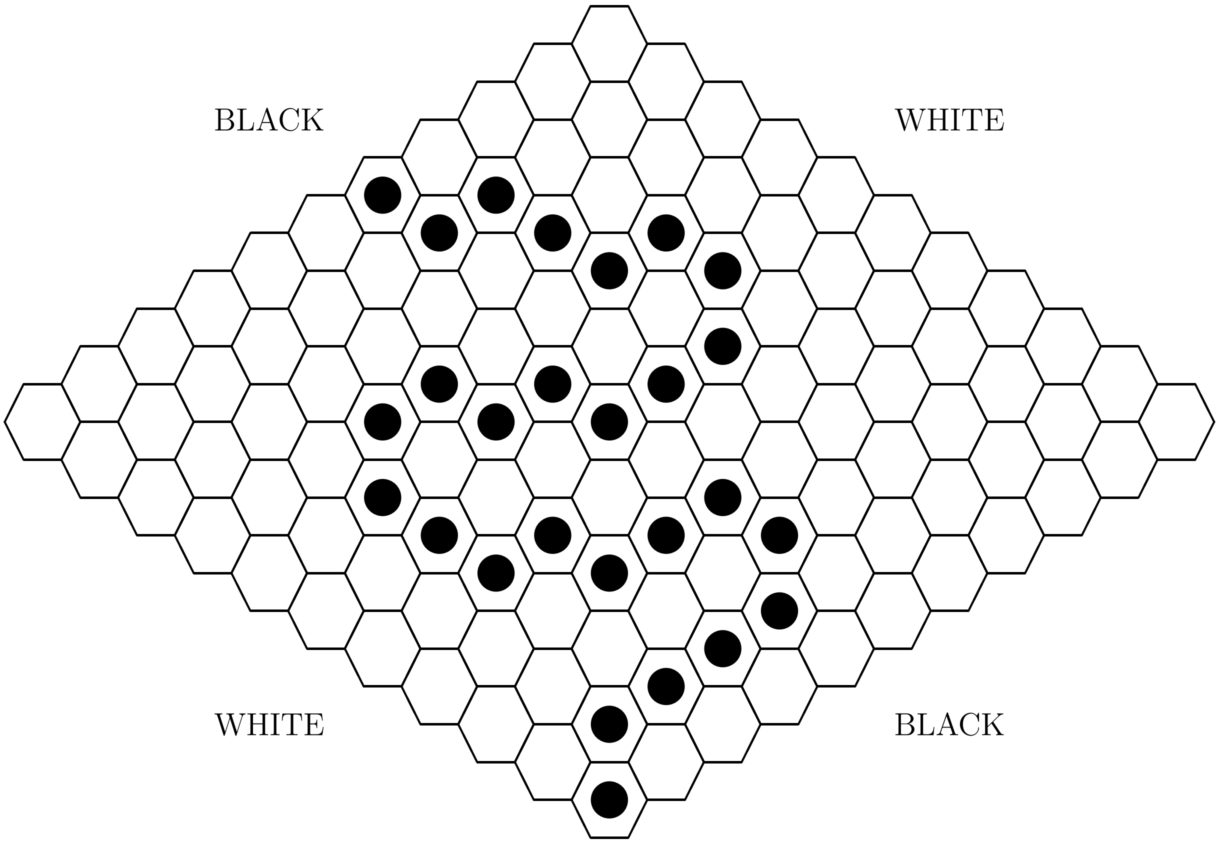

Figure 14: An Example of a Path in Hex

20 HEX II

Hex II is played on the board shown in Figure 15. Its rules are the same as Hex.

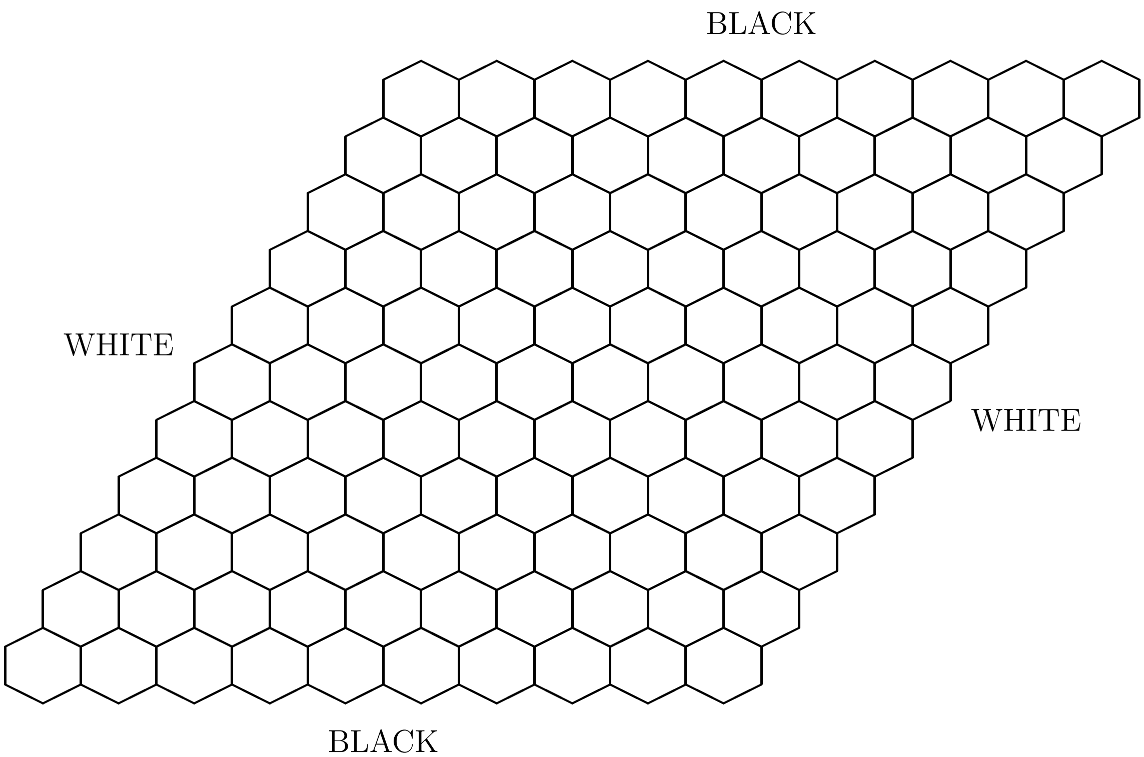

Figure 15: The Game of Hex II

21 HIT-AND-RUN

The game of Hit-and-Run is played on a lattice. A five by five lattice seems to be an interesting size (see Figure 16). Players take turns connecting two

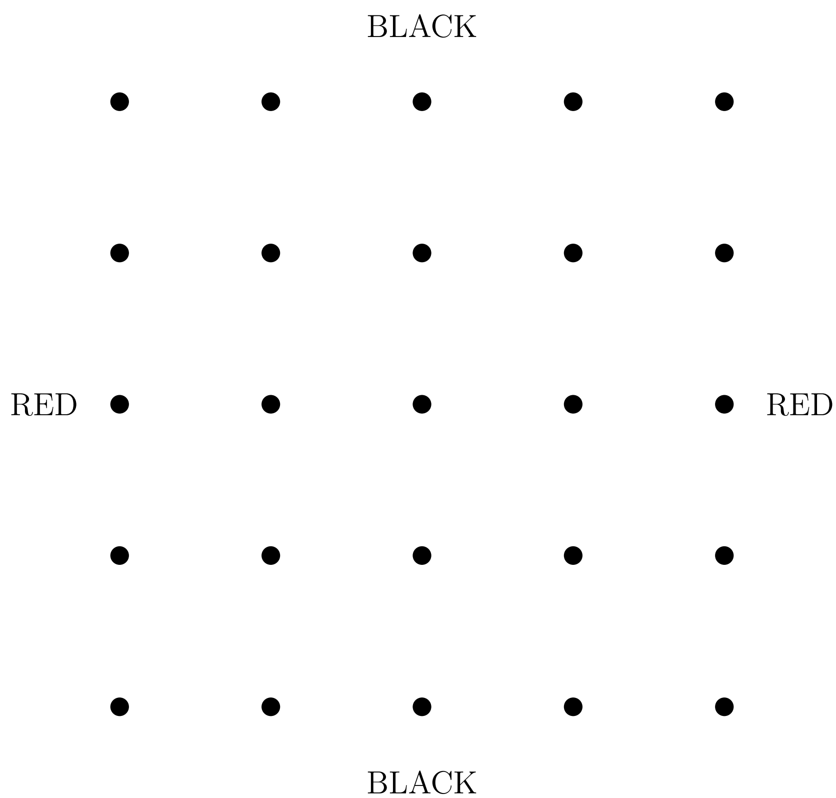

Figure 16: The Game of Hit-and-Run

adjacent points, either horizontally or vertically. One player uses a black pencil; the other uses red. Black's object is to construct a path connecting the upper and lower sides of the board, Red's is to construct a path connecting the left and right sides. (Opposing paths may cross each other at a right angle.) The first player to build his path wins. See Figure 17 for a completed game in which Black has won. The game is called Hit-and-Run because a move can block an opponent's path (a hit) and at the same time extend the player's own path (a run).

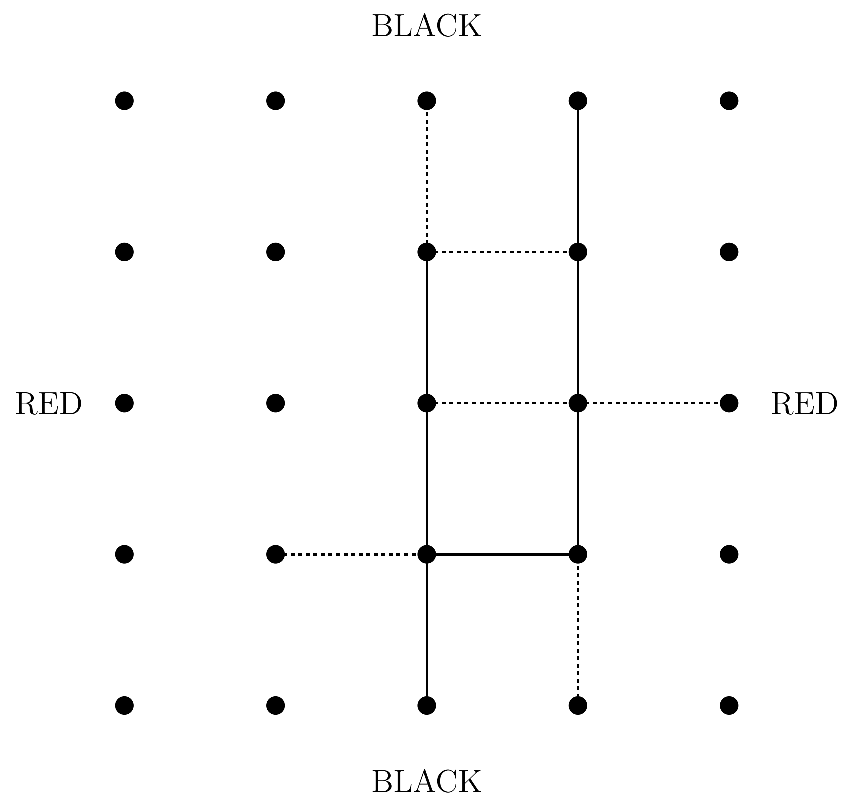

Figure 17: A Completed Game of Hit-and-Run

22 CHOMP

Chomp can be played with a supply of counters (see Figure 18) or with O's or X's on a sheet of paper. The counters are arranged in a rectangular

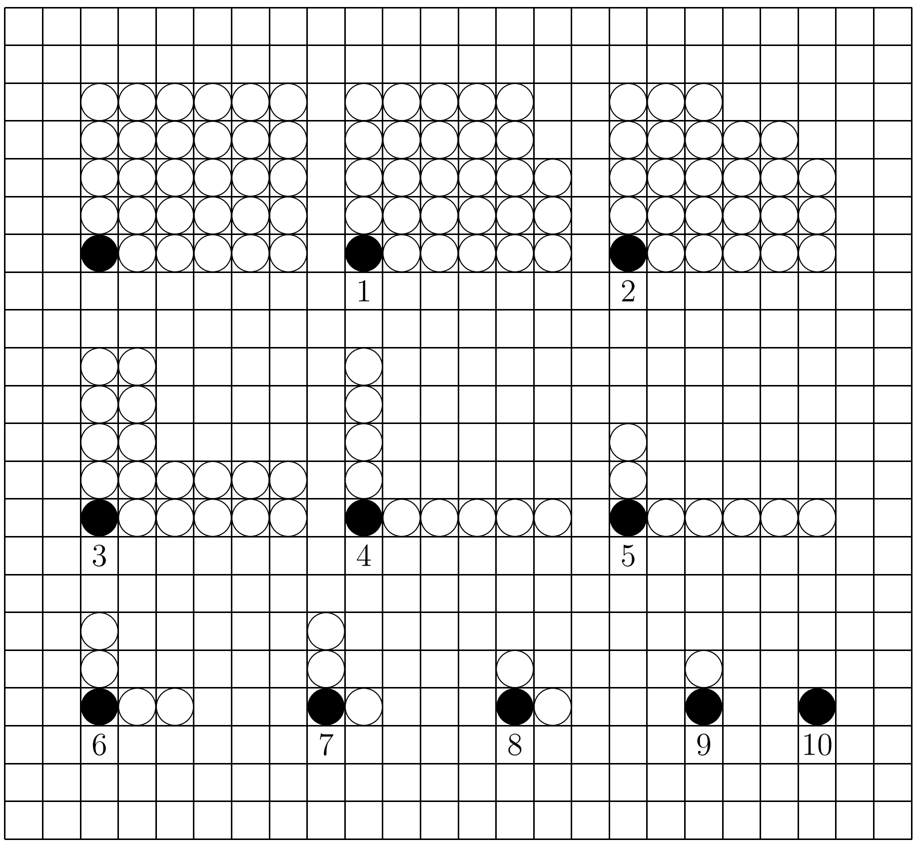

Figure 18: Chomp on a 5×6 Field

formation. Two players take turns removing counters as follows. Any counter is selected. Imagine that this counter is inside the vertex of a right angle through the field, the base of the angle extending east below the counter's row and its other side extending vertically north along the left side of the counter's column. All counters inside the right angle are removed. This constitutes a move. It is as though the field were a cracker and a right-angled bite were taken from it by jaws approaching the cracker from the northeast.

The object of the game is to force your opponent to chomp the poison counter at the lower left corner of the array (black counter).

23 THE DIVISOR GAME

Two players agree on any positive integer N. A list is made of all the divisors (including N and 1); then players take turns crossing out a divisor and all *its* divisors. The person forced to take N loses.

24 SPROUTS

The game of Sprouts begins with n spots on a sheet of paper. Even with as few as three spots, Sprouts is more difficult to analyze than Ticktacktoe, so that it is best for beginners to play with no more than three or four initial spots. A move consists of drawing a curve that joins one spot to another or to itself and then placing a new spot anywhere along the curve. These restrictions must be observed:

1. The curve may have any shape but it must not cross itself, cross a previously drawn curve or pass through a previously made spot.

2. No spot may have more than three curves emanating from it.

Players take turns drawing curves. In normal sprouts, the recommended form of play, the winner is the last person able to play. In misère sprouts the first person unable to play is the winner.

25 BRUSSELS SPROUTS

Brussels sprouts begins with n crosses instead of spots. A move consists of extending any arm of any cross into a curve that ends at the free arm of any other cross or the same cross; then a crossbar is drawn anywhere along the curve to create a new cross. Two arms of the new cross will, of course, be dead, since no arm may be used twice. As in sprouts, no curve may cross itself or cross a previously drawn curve, nor may it go through a previously made cross. As in sprouts, the winner of the normal game is the last person to play and the winner of the misère game is the first person who cannot play.