

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

a  b  c  d  e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

**GOOD LUCK!**

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For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

**Multiple Choice Questions**

Show all your work on the page where the question appears.  
Clearly mark your answer both on the cover page on this exam  
and in the corresponding questions that follow.

1. Solve for  $b$  in  $2(9 - \sqrt{b}) = 16$ .

Possibilities:

- (a)  $b = 1$
- (b)  $b = 10$
- (c)  $b = -7$
- (d)  $b = \frac{13}{2}$
- (e) No solution

*\*unwrapping\**

multiply by 2 → divide by 2  $\frac{2(9 - \sqrt{b})}{2} = \frac{16}{2}$

add 9 → subtract 9  $9 - \sqrt{b} = 8$

negate → multiply by -1  $-9 - \sqrt{b} = -1$

sq. root → square  $(\sqrt{b})^2 = (1)^2$

$b = 1$

*\*wrapping\**

2. Find the  $y$ -intercept(s) of the graph of  $y - 17 = x^2 - 8x - 2$ .

Possibilities:

- (a) (3, 15) and (5, 15)
- (b) (3, 0) only
- (c) (0, 15) only
- (d) (3, 0) and (5, 0)
- (e) (5, 0) only

*y-intercepts occur when  $x=0$*

$(0, 15)$

$y = 15$

$y - 17 = x^2 - 8x - 2$

$y - 17 = 0^2 - 8(0) - 2$

$y - 17 = 0 - 0 - 2$

$y - 17 = -2$

$+17 \quad +17$

3. Solve for  $x$  in  $3 + |1 - x| = 5$ .

Possibilities:

- (a) 7 and -1
- (b) 7 only
- (c) -1 only
- (d) 3 only
- (e) -1 and 3

*\*isolate absolute value expression first\**

$3 + |1 - x| = 5$

$-3 \quad -3$

$|1 - x| = 2$

*\*expression inside absolute value can be + or -\**

$1 - x = 2$

$-x = 1$

$x = -1$

$1 - x = -2$

$-x = -3$

$x = 3$

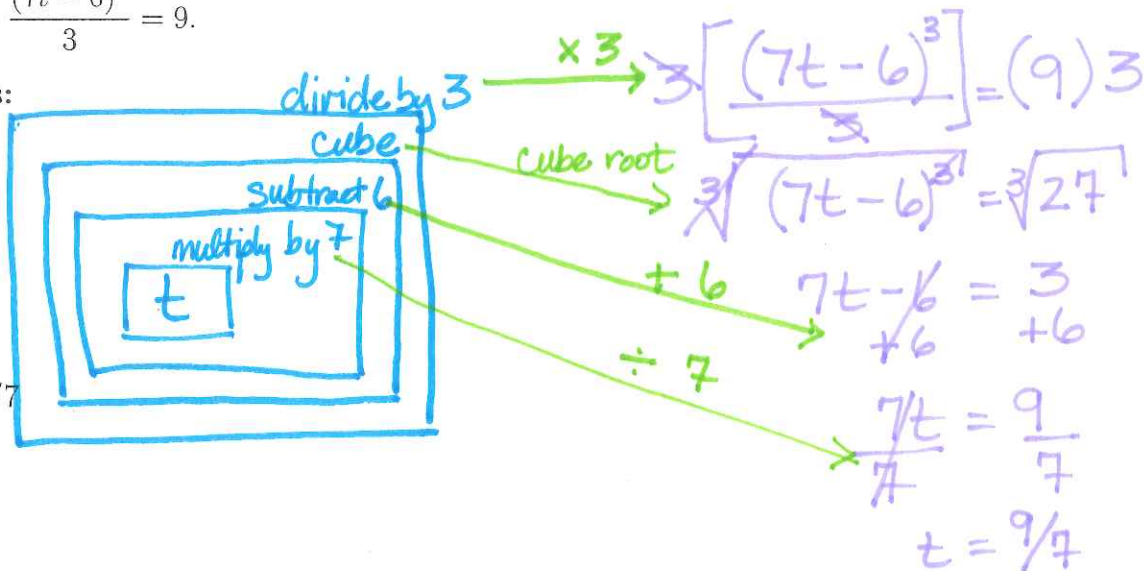
*\*solve both possibilities for x\**

2

4. Solve for  $t$  in  $\frac{(7t-6)^3}{3} = 9$ .

Possibilities:

- (a)  $\frac{7}{9}$
- (b) 61731
- (c)  $\frac{9}{7}$**
- (d)  $6 \pm \sqrt{27}/7$
- (e)  $\frac{15}{7}$



5. The point (7, 4) is on the graph of which of the following equations?

Possibilities:

- (a)  $x = y - 3$
- (b)  $xy + 28 = xy + 16$
- (c)  $4x + 28 = 4y + 28$
- (d)  $xy = 0$
- (e)  $4x + 28 = xy + 28$**

\*points on graphs are solutions to equations!

(a)  $7 \stackrel{?}{=} 4 - 3$   
 $7 \neq 1$

(b)  $7(4) + 28 \stackrel{?}{=} 7(4) + 16$   
 $28 + 28 \neq 28 + 16$

(c)  $4(7) + 28 \stackrel{?}{=} 4(4) + 28$   
 $28 + 28 \neq 16 + 28$

(d)  $7(4) \stackrel{?}{=} 0$   
 $28 \neq 0$

(e)  $4(7) + 28 \stackrel{?}{=} 7(4) + 28$   
 $28 + 28 \stackrel{?}{=} 28 + 28$

6. The graph of  $x^2 + y^2 - 14x - 8y + 61 = 0$  is a circle. Find its center and its radius.

\*standard form for equation of circle:  $(x-h)^2 + (y-k)^2 = r^2$

Possibilities:

- (a) Radius: 4      Center: (14, 8)
- (b) Radius:  $\sqrt{61}$       Center: (7, 4)
- (c) Radius: 2      Center: (7, 4)**
- (d) Radius:  $\sqrt{61}$       Center: (-7, -4)
- (e) Radius: 2      Center: (-7, -4)

(h, k) is center & r is radius

$$x^2 - 14x + 49 - 49 + y^2 - 8y + 16 - 16 + 61 = 0$$

$$(x-7)^2 + (y-4)^2 - 49 - 16 + 61 = 0$$

$$(x-7)^2 + (y-4)^2 - 4 = 0$$

$$(x-7)^2 + (y-4)^2 = 4$$

$$(x-7)^2 + (y-4)^2 = 2^2$$

center: (7, 4)      radius: 2

7. How many distinct, real solutions does each equation have?

(I)  $4x^2 + 9x + 5 = 0$

(II)  $7x^2 + 2x + 3 = 0$

Possibilities:  $*b^2 - 4ac > 0$  2 distinct, real     $*b^2 - 4ac = 0$  1 distinct, real     $*b^2 - 4ac < 0$  0 distinct, real

- (a) (I) has 2 distinct, real solutions; and (II) has 1 distinct, real solution
- (b) (I) has 1 distinct, real solution; and (II) has 2 distinct, real solutions
- (c) (I) has 0 distinct, real solutions; and (II) has 1 distinct, real solution
- (d) (I) has 0 distinct, real solutions; and (II) has 2 distinct, real solutions
- (e) (I) has 2 distinct, real solutions; and (II) has 0 distinct, real solutions**

(I)  $a = 4$   
 $b = 9$   
 $c = 5$

$b^2 - 4ac$   
 $9^2 - 4(4)(5)$   
 $81 - 80$

$1 > 0 \Rightarrow 2 \text{ distinct, real}$

(II)  $a = 7$   
 $b = 2$   
 $c = 3$

$b^2 - 4ac$   
 $2^2 - 4(7)(3)$   
 $4 - 84$

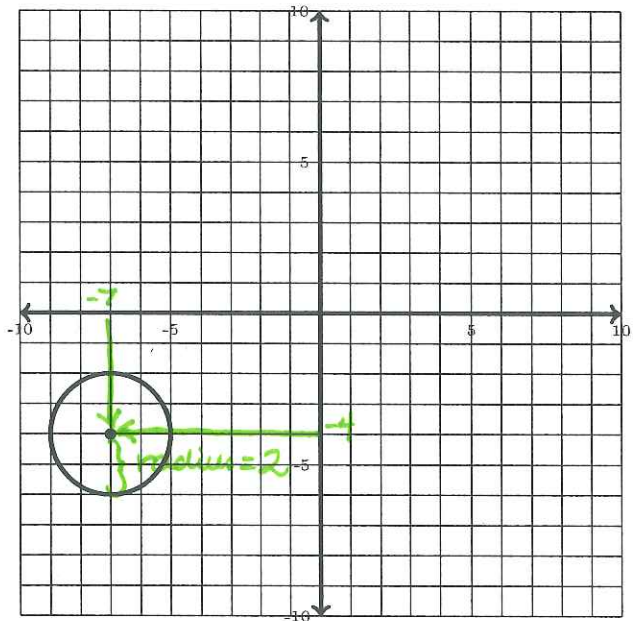
$-80 < 0 \Rightarrow 0 \text{ distinct, real}$

8. Find an equation for the circle shown below:

$* \text{standard form for circle}$   
 $(x-h)^2 + (y-k)^2 = r^2$

Possibilities:

- (a)  $(x + 14)^2 + (y - 8)^2 = -4$
- (b)  $(x - 7)^2 + (y + 4)^2 = 2$
- (c)  $(x - 7)^2 + (y - 4)^2 = 4$
- (d)  $(x + 7)^2 + (y + 4)^2 = 4$**
- (e)  $(x + 7)^2 + (y - 4)^2 = 2$



center:  
 $(-7, -4)$   
 radius:  
 $2$

$[x - (-7)]^2 + [y - (-4)]^2 = 2^2$

$(x + 7)^2 + (y + 4)^2 = 4$

9. Find all distinct, real solutions  $x$  to  $\sqrt{6-x} = x-4$

*\* Squaring both sides of an equation can introduce extraneous solutions.*

Possibilities:

- (a) 6 only
- (b) 5 only**
- (c) 2 and 5
- (d) 2 only
- (e) 6 and -4

$$(\sqrt{6-x})^2 = (x-4)^2$$

$$\begin{array}{r} 6-x \\ -6+x \end{array} = \begin{array}{r} x^2-8x+16 \\ +x-6 \end{array}$$

$$0 = x^2 - 7x + 10$$

$$0 = (x-5)(x-2)$$

$$\begin{array}{l} x-5=0 \\ x=5 \end{array} \quad \begin{array}{l} x-2=0 \\ x=2 \end{array}$$

**CHECK!**

$$\boxed{x=5} \quad \begin{array}{l} \sqrt{6-5} \stackrel{?}{=} 5-4 \\ \sqrt{1} \stackrel{?}{=} 1 \end{array}$$

~~$$x=2 \quad \begin{array}{l} \sqrt{6-2} \stackrel{?}{=} 2-4 \\ \sqrt{4} \stackrel{?}{=} -2 \\ 2 \neq -2 \end{array}$$~~

*\* set each factor = 0 and solve*

*\* don't forget to check solutions!*

10. Find all distinct, real solutions  $x$  to  $3x = x^5$ .

*\* zero product property a method for finding solutions to non-quadratic equations.*

Possibilities:

- (a)  $x = \sqrt[5]{3}$  only
- (b)  $x = 0$  and  $x = \sqrt[5]{3}$
- (c)  $x = 0$  and  $x = \pm\sqrt[4]{3}$**
- (d)  $x = \pm\sqrt[4]{3}$  and  $x = \sqrt[5]{3}$
- (e)  $x = \pm\sqrt[4]{3}$  only

$$\begin{array}{r} 3x \\ -3x \end{array} = \begin{array}{r} x^5 \\ -3x \end{array}$$

$$0 = x^5 - 3x$$

$$0 = x(x^4 - 3)$$

$$\begin{array}{l} x=0 \\ x^4-3=0 \end{array}$$

$$x^4 = 3$$

$$x = \pm\sqrt[4]{3}$$

11. Find all distinct, real solutions  $x$  to  $x^2 + 7x + 5 = 0$ .

*\* Quadratic formula appropriate since equation does not factor nicely.*

Possibilities:

- (a)  $\frac{-7 \pm \sqrt{29}}{2}$**
- (b)  $\frac{-7 \pm \sqrt{69}}{2}$
- (c)  $\frac{\pm 7 - \sqrt{29}}{2}$
- (d)  $\frac{\pm 7 - \sqrt{69}}{2}$
- (e) No solution

$$x^2 + 7x + 5 = 0$$

$$a=1, b=7, c=5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 - 20}}{2}$$

$$x = \frac{-7 \pm \sqrt{29}}{2}$$

12. How many distinct, real solutions  $x$  does  $\frac{3}{x-8} + \frac{1}{x-6} = \frac{5}{x^2 - 14x + 48}$  have?

\* clearing denominators can introduce extraneous solutions!

Possibilities:

- (a) 4 solutions
- (b) No solutions
- (c) 3 solutions
- (d) 2 solutions
- (e) 1 solution

$$\left[ \frac{3}{x-8} + \frac{1}{x-6} = \frac{5}{(x-8)(x-6)} \right] \begin{matrix} (x-8)(x-6) \\ \text{multiply by LCD} \\ \text{to clear denominators} \end{matrix}$$

$$(x-8)(x-6)\left(\frac{3}{x-8}\right) + (x-8)(x-6)\left(\frac{1}{x-6}\right) = (x-8)(x-6)\left[\frac{5}{(x-8)(x-6)}\right]$$

$$3(x-6) + 1(x-8) = 5$$

$$3x - 18 + x - 8 = 5$$

$$4x - 26 = 5$$

$$4x = 31$$

$$x = \frac{31}{4}$$

one viable solution since  $x \neq 8$  or  $x \neq 6$

13. Find an equation for the line through the points  $(7, 4)$  and  $(2, 6)$ .

\* find slope & use point-slope form

Possibilities:

- (a)  $y + 4 = -\frac{5}{2}(x + 7)$
- (b)  $y - 4 = -\frac{2}{5}(x - 7)$
- (c)  $y + 4 = -\frac{2}{5}(x + 7)$
- (d)  $y - 4 = -\frac{5}{2}(x - 7)$
- (e)  $y = \frac{5}{2}(x - 7) - 4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 - 4}{2 - 7}$$

$$m = \frac{2}{-5}$$

$$m = -\frac{2}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{2}{5}(x - 7)$$

14. Rewrite the expression  $x^2 - 2x + 5$  by completing the square.

\* complete square by adding and subtracting  $\left(\frac{b}{2}\right)^2$

Possibilities:

- (a)  $(x - 1)^2 + 4$
- (b)  $(x + 2)^2 - 5$
- (c)  $(x + 1)^2 - 4$
- (d)  $(x - 2)^2 + 5$
- (e)  $(x + 1)^2 - 5$

$$x^2 - 2x + \underbrace{1}_{\left(\frac{-2}{2}\right)^2} - \underbrace{1}_{\left(\frac{-2}{2}\right)^2} + 5$$

$$(x - 1)^2 + 4$$



18. Solve the equation  $6x^2 + 104xy = 3$  for  $y$  in terms of  $x$

*\* isolate "y" term first and then isolate "y" variable*

Possibilities:

(a)  $y = \frac{104x}{6x^2 - 3}$

(b)  $y = 3 - 6x^2 - 104x$

(c)  $y = \frac{-104 \pm \sqrt{10888}}{12}$

(d)  $y = \frac{3 - 6x^2}{104x}$

(e)  $y = \frac{6x^2 - 3}{104x}$

$$\begin{aligned}
 6x^2 + 104xy &= 3 \\
 -6x^2 & \quad -6x^2 \\
 \hline
 104xy &= 3 - 6x^2 \\
 \frac{104xy}{104x} &= \frac{3 - 6x^2}{104x} \\
 y &= \frac{3 - 6x^2}{104x}
 \end{aligned}$$

19. Find all distinct, real solutions  $x$  to  $x^{10} - 8x^5 + 12 = 0$

*\* quasi-quadratic can be solved with u-substitution method.*

Possibilities:

(a)  $x = 6^5$  and  $x = 2^5$

(b)  $x = \sqrt[5]{6}$  and  $x = \sqrt[5]{2}$

(c)  $x = 2$  only

(d)  $x = 6$  only

(e)  $x = 6$  and  $x = 2$

$$\begin{aligned}
 u &= x^5 \\
 u^2 - 8u + 12 &= 0 \\
 (u-6)(u-2) &= 0 \\
 u-6 &= 0 & \quad u-2 &= 0 \\
 u &= 6 & \quad u &= 2 \\
 x^5 &= 6 & \quad x^5 &= 2 \\
 x &= \sqrt[5]{6} & \quad x &= \sqrt[5]{2}
 \end{aligned}$$

*← solve for "u" & substitute to solve for "x"*

20. What is the distance between  $(-3, 1)$  and  $(5, 8)$ ?

*\* distance =  $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$*

(a) 7

(b) 5

(c) 8

(d)  $\sqrt{113}$

(e)  $\sqrt{15}$

$$\begin{aligned}
 \text{distance} &= \sqrt{(-3-5)^2 + (1-8)^2} \\
 &= \sqrt{(-8)^2 + (-7)^2} \\
 &= \sqrt{64 + 49} \\
 &= \sqrt{113}
 \end{aligned}$$