

8 Inequalities

Concepts:

- Equivalent Inequalities
- Linear and Nonlinear Inequalities
- Absolute Value Inequalities

(Sections 4.6 and 1.1)

8.1 Equivalent Inequalities

Definition 8.1

Two inequalities are **equivalent** if they have the same solution set.

Operations that Produce Equivalent Inequalities.

- Add or Subtract the same value on both sides of the inequality.
- Multiply or Divide by the same **positive value** on both sides of the inequality.
- Multiply or Divide by the same **negative value** on both sides of the inequality **AND change the direction of the inequality**.

Example 8.2

For each pair of inequalities, determine if the two inequalities are equivalent.

- $x^2 + 2x \leq 5$ and $x^2 + 2x - 5 \leq 0$.
- $6 - x < 5$ and $x > 1$
- $\frac{x}{x+2} > 3$ and $\frac{x}{x+2} - 3 > 0$
- $\frac{x}{x+2} > 3$ and $x > 3(x+2)$

You should **NEVER** multiply both sides of an inequality by an expression involving x that you don't know the sign of. Why?

8.2 Solving a Linear or Nonlinear Inequality

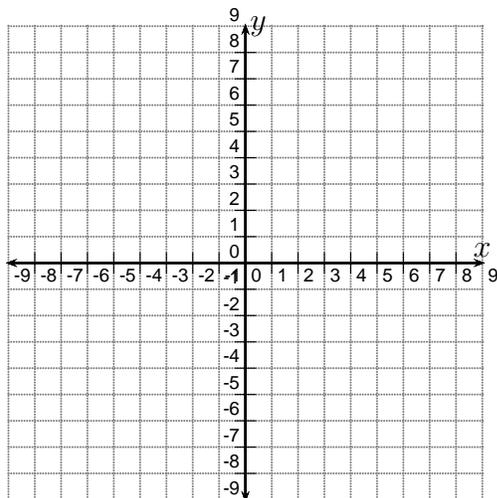
Example 8.3 (Linear Inequality)

Solve the inequality $5x + 3 \leq 6 - 7x$. Write the solution set in interval notation.

Note 8.4

There are many ways to solve this inequality algebraically. We will begin by using addition and subtraction to move all the nonzero quantities to one side. This is not necessary for this inequality, but it will help us to understand the process needed for solving more complicated inequalities.

Graph the equations $y = 5x + 3$ and $y = 6 - 7x$. How can you approximate the solutions of an inequality graphically?



Example 8.7 (Quadratic Inequality)

Solve the inequality below. Be sure to write your answer in interval notation.

$$x^2 + 2x > 8$$

Example 8.8 (Rational Inequality)

Solve the inequality below. Be sure to write your answer in interval notation.

$$\frac{2}{x+3} \leq \frac{4}{x-1}$$

Example 8.9 (Inequality Application)

A computer store has determined that the cost C of ordering and storing x laser printers is given by $C = 2x + \frac{300,000}{x}$. If the delivery truck can bring at most 450 printers per order, how many printers should be ordered at a time to keep the cost below \$1600?

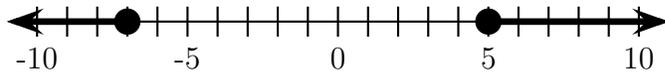
8.3 Absolute Value Inequalities

Number lines can be really insightful when working with absolute value equations and inequalities. Recall that we think of the absolute value as a distance.

Example 8.10 (Another Distance Example)

Solve $|x + 3| > 5$ geometrically. Be sure to write your answer in interval notation.

Example 8.11 (Another Distance Example)



(a) Write a distance sentence that corresponds to this number line.

(b) Write an absolute value equation or inequality that corresponds to this number line.

Example 8.12 (The Algebraic Approach to Absolute Values)

Solve each inequality algebraically.

(a) $|2x + 3| \geq 7$

(b) $|x + 2| + 1 < 3$