Name: ____ **Section:** ________

MA 109 Spring 2014

Exam 2 March 12, 2014

Directions:

- Do not remove this page—you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.
- The exam consists of multiple choice and short answer questions. Record your answers on this page by filling in the appropriate selection, for example:

  A B C D E.

- The exam is out of 100 total points: 5 points for each of 20 questions. **Only** this front page will be graded and **no partial credit** will be awarded. It is recommended that you check your work!

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E

Bonus questions for exam 1

21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E

For grading use:

Total			**Total**	
	(out of 100 pts)			**(out of 25 pts)**

Name: ____ **Section:** __________

Multiple Choice: Show your work in the space below and shade the correct answer on the front page for each of the following.

1. Find the slope of the line through the points (2,1), (-1,3).

$x_1\ y_1 \quad x_2\ y_2$

Choices:

(a) $\boxed{\frac{-2}{3}}$

(b) $\frac{-3}{2}$

(c) $\frac{4}{3}$

(d) $\frac{-4}{3}$

(e) $\frac{3}{2}$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-1-2} = \frac{2}{-3} = -\frac{2}{3}$$

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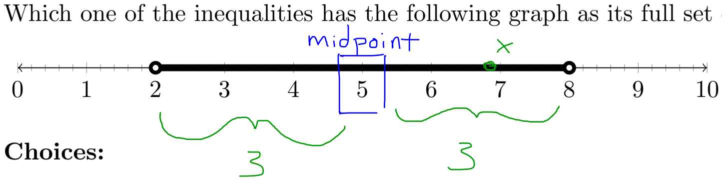
2. Which one of the inequalities has the following graph as its full set of solutions?



$$\text{midpoint} = \frac{8+2}{2} = \frac{10}{2} = 5$$

Choices:

(a) $|x-5| \leq 3$

(b) $\boxed{|x-5| < 3}$

(c) $|x-2| > 6$

(d) $|x-8| < 6$

(e) $|x+5| < 3$

The distance from x to 5 | is less than | 3

$|x-5| \quad < \quad 3$

So $|x-5| < 3$

---

3. Find the equation of the line through the point (-2,1) with slope -3.

$x_1\ y_1$

Choices:

(a) $y = 3x + 3$

(b) $y = -3x + 1$

(c) $\boxed{y = -3x - 5}$

(d) $y = -2x - 3$

(e) $y = 2x + 3$

Point Slope form: $y - y_1 = m(x - x_1)$

$y - 1 = -3(x - (-2))$ Simplify

$y - 1 = -3(x + 2)$ Distribute -3

$y - 1 = -3x - 6$ Add 1

$y - 1 + 1 = -3x - 6 + 1$ Simplify

$\boxed{y = -3x - 5}$

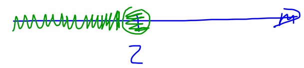
---

4. Solve the inequality $5x - 2 \le x + 6$.

Choices:

(a) $\boxed{(-\infty, 2]}$
(b) $(-\infty, 2)$
(c) $[-2, \infty)$
(d) $[2, \infty)$
(e) $[4, \infty)$

$5x - 2 \le x + 6$ Add 2 (same)
$5x - 2 + 2 \le x + 6 + 2$ Simplify
$5x \le x + 8$ Subtract x (same)
$5x - x \le x + 8 - x$ Simplify
$4x \le 8$ Divide by 4 (same)
$\frac{\cancel{4}x}{\cancel{4}} \le \frac{8}{4}$ Simplify
$x \le 2$



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5. Use a graphing calculator to approximate the real solutions to the equation below.

$$x^3 + 2x - 5 = 0$$

Choices:

Using your graphing calculator plug into $y = x^3 + 2x - 5$ and calculate the zero.

(a) $x \approx -2.0946$
(b) $\boxed{x \approx 1.3283}$
(c) $x \approx 1.2763$
(d) $x \approx 1.4894$
(e) $x \approx 1.7099$

---

6. Determine all solutions to the system $\begin{cases} x + y = 3 \\ x - y = -2 \end{cases}$

Choices:

Elimination Method:

(a) $(1, 2)$
(b) $(6, 8)$
(c) $(1, 3)$
(d) $\boxed{\left(\frac{1}{2}, \frac{5}{2}\right)}$
(e) $\left(\frac{3}{2}, \frac{2}{5}\right)$

$x + y = 3$
$x - y = -2$
$2x + 0y = 1$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{1}{2}$
$x = 1/2$

backsubstitute $x = 1/2$

$\frac{1}{2} + y = 3$ Subtract $\frac{1}{2}$
$\frac{1}{2} + y - \frac{1}{2} = 3 - \frac{1}{2}$ Simplify
$y = \frac{3}{1} \cdot \frac{2}{2} - \frac{1}{2}$ Simplify
$y = \frac{6}{2} - \frac{1}{2} = \frac{6-1}{2} = \frac{5}{2}$

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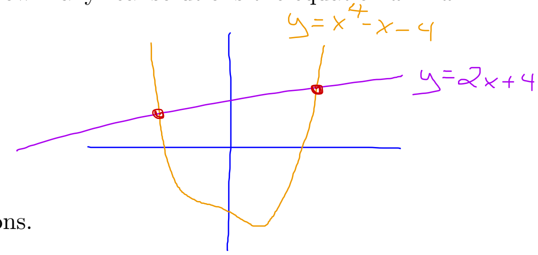
7. Use a graphing calculator to determine how many real solutions the equation $x^4 - x - 4 = 2x + 4$ has.



Choices:

(a) Exactly four real solutions.
(b) Exactly one real solution.
$\boxed{\text{(c) Exactly two real solutions.}}$
(d) The equation has no real solutions.
(e) Exactly three real solutions.

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8. If a student has exam scores of 88, 62, and 79 on his first three exams, what does he need on the fourth exam to have an average of 80.

Choices:

$\boxed{\text{(a) } 91}$
(b) 85
(c) 95
(d) 100
(e) The student can not obtain an average of 80.

Let x be the fourth exam grade

$\frac{88+62+79+x}{4} = 80$ Simplify

$\frac{229+x}{4} = 80$ Multiply by 4

$\cancel{4}\,\frac{229+x}{\cancel{4}} = 4 \cdot 80$ Simplify

$229 + x = 320$ Subtract 229

$229 + x - 229 = 320 - 229$ Simplify

$x = 91$

Conclusion: The student will need to earn a fourth exam score of 91 to have an average of 80

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9. Which one of the following equations can not be solved algebraically and so must be solved graphically?

Choices:

(a) $\frac{1}{x+1} - \frac{5}{x-3} = 10$ Multiply by $(x+1)(x-3)$ turns into a quadratic
(b) $x^2 - x + 1 = 0$ Quadratic
(c) $3x^5 - 1 = 0$ Power equation $x^n = c$
(d) $\sqrt{x-2} = 5x$ Undo radical yields a quadratic
$\boxed{\text{(e)}}$ $x^3 + x = x^2 - 1$

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10. Which statement best describes the slope of a line?

Choices:

(a) The slope of a line changes from point to point. The slope is independent of the points

(b) The slopes of two ~~perpendicular~~ parallel lines are the same.

(c) A vertical line has ~~a~~ an undefined slope ~~of 0~~.

(d) The slope of a line is calculated by the ratio ~~run~~ rise over ~~rise~~ run.

$\boxed{\text{(e)}}$ The slope of a line represents the rate of change of the line.

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11. How many liters of a 10% solution of acid must be mixed with 20 liters of a 20% solution of acid to produce an 18% solution of acid?

Choices:

(a) $\frac{7}{3}$ liters.

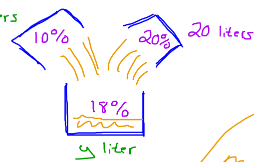
(b) The final solution can not be obtained.

(c) $\frac{5}{2}$ liters.

$\boxed{\text{(d) 5 liters.}}$

(e) 50 liters.

Need to solve for x → X liters



Equations:

$x + 20 = y$

$.10x + .20(20) = .18y$

mult by $-.18$

$-.18x - 3.6 = -.18y$

$.10x + 4 = .18y$

$-0.08x + 0.4 = 0$

$\frac{0.4}{0.08} = \frac{\cancel{0.08}x}{\cancel{0.08}}$

$x = \frac{.4}{0.08} = 5$ liters

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12. Find the slope of the line $2x + 4y = 10$.

Choices:

$\boxed{\text{(a) } \frac{-1}{2}}$

(b) 2

(c) $\frac{1}{2}$

(d) -1

(e) -2

Need to put into slope intercept form $y = mx + b$

$2x + 4y = 10$ Subtract $2x$

$2x + 4y - 2x = 10 - 2x$ Simplify

$4y = -2x + 10$ Divide by 4

$\frac{\cancel{4}y}{\cancel{4}} = \frac{-2x + 10}{4}$ Simplify

$y = \frac{-2}{4}x + \frac{10}{4}$

$y = \left(\frac{-1}{2}\right)x + \left(\frac{5}{2}\right)$

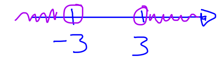
slope

y-coordinate of the y-intercept

13. Which is the full set of solutions to $|x-5| > 3$?

Let $w = x - 5$ then $|w| > 3$

so



Choices:

(a) $(-\infty, -2] \cup [8, \infty)$

(b) $(5, 8)$

(c) $\boxed{(-\infty, 2) \cup (8, \infty)}$

(d) $[2, 8]$

(e) $(-\infty, 5)$

$w < -3$ and $w > 3$

$x - 5 < -3$

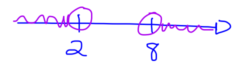
$x - 5 + 5 < -3 + 5$

$x < 2$

$x - 5 > 3$

$x - 5 + 5 > 3 + 5$

$x > 8$



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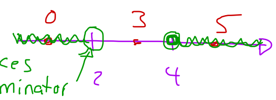
14. Which is the full set of solutions to $\dfrac{x-4}{x-2} \geq 0$?

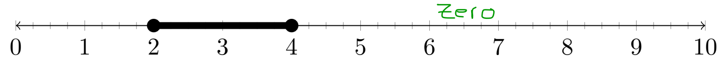
Critical Numbers are $x = 4$; $x = 2$

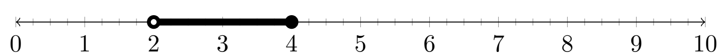


makes denominator zero

Test Points	$x-4$	$x-2$	Sign
0	$-$	$-$	$+$
3	$-$	$+$	$-$
5	$+$	$+$	$+$

Choices:

(a) 

(b) 

(c) 

(d) 

(e) 

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15. Given a system of two equations where the graph of one equation is a line and the other graph is a circle, what are the possible number of solutions to the system?

Choices:

(a) Always two solutions.

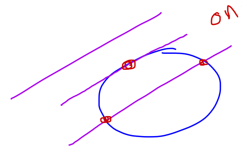
(b) One or two solutions only.

(c) $\boxed{\text{One, two, or no solutions.}}$

(d) Any number of solutions depending on the graphs.

(e) Two solutions or no solution only.

no solutions
one solution
two solutions



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16. Given the two lines $y = 2x + 3$ and $y = -2x + 1$, which one of the following statements is true?

$m = 2$

slope $= -2$

Choices:

$\parallel$ $m = 2$

$\perp$ $m = -1/2$

So ~~not~~ parallel or perpendicular

also different lines

x (a) The lines are parallel.

x (b) The lines are perpendicular.

x (c) The two lines are the same.

$\boxed{\text{(d)}}$ The two lines intersect at exactly one point.

(e) The two lines intersect at more than one point.

Two lines cannot intersect in more than one point.

---

17. Solve the inequality

$$(x - 3)(x + 5) < 0.$$

Critical numbers are $x = 3$ and $x = -5$

Choices:

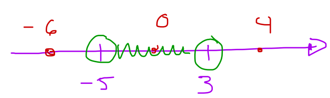
(a) $(-\infty, 3] \cup [5, \infty)$

(b) $(-\infty, -3) \cup [5, \infty)$

$\boxed{\text{(c)} \quad (-5, 3)}$

(d) $[-5, 3]$

(e) $(5, \infty)$



Test Points	$x - 3$	$x + 5$	sign	
-6	−	−	+	x
0	−	+	−	✓
4	+	+	+	x

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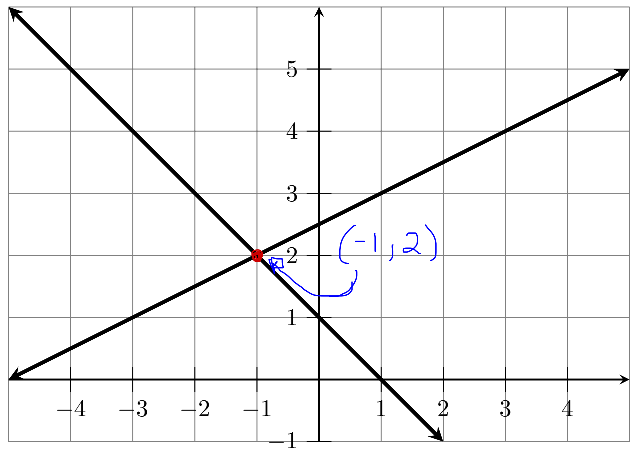
18. Suppose you are given a system of equations whose graphs are shown in the picture below. Determine an approximate solution to this system.



Choices:

(a) $(-1, 2)$

(b) $(-\infty, -1]$

(c) $(2, \infty)$

(d) $(-2, 1)$

(e) $(1, 2)$

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19. If the graph of $y^2 + 3x - 2 = 0$ is to be displayed on a calculator, which of the following statements best describes the procedure?

$y^2 = -3x + 2$

$y = \pm\sqrt{-3x+2}$

Choices:

(a) Solve for y and enter it into the calculator as a single equation.

(b) Solve for x and enter it into the calculator as a single equation.

(c) Solve for y and enter it in the calculator as two separate equations.

(d) Enter the equation directly into the calculator.

(e) The graph of the equation can not be displayed on a graphing calculator.

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20. Determine all solutions to the system $\begin{cases} x^2 + y = 3 \\ y - 2x = 0 \end{cases}$

Substitution Method

$y - 2x = 0 \xrightarrow{\text{solve for } y} y = 2x$

Substitute into equation one

$x^2 + 2x = 3$

$x^2 + 2x - 3 = 0$

$x^2 + 3x - 1x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x-1)(x+3) = 0$

$x - 1 = 0 \quad x = 1$

$x + 3 = 0 \quad x = -3$

If $x = 1$ then $y = 2(1) = 2$

so $(1,2)$ is a solution

If $x = -3$ then $y = 2(-3) = -6$

so $(-3,-6)$ is a solution

Choices:

(a) The system has no solution.

(b) $(-3, -6)$ only.

(c) $(1, 1)$ only.

(d) $(1, 2)$ and $(-1, -1)$.

(e) $\boxed{(-3, -6) \text{ and } (1, 2).}$

Bonus Multiple Choice: Show your work in the space below and shade the correct answer on the front page for each of the follow

21. Solve the following equation for x.

$$5x^2 - 3x = 1$$

$5x^2 - 3x - 1 = 0$

$a = 5$

$b = -3$

$c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{9 + 20}}{10} = \frac{3 \pm \sqrt{29}}{10}$$

Choices:

(a) $\boxed{\dfrac{3 \pm \sqrt{29}}{10}}$

(b) The equation has no real solution.

(c) $\dfrac{\pm\sqrt{2}}{5}$

(d) $\dfrac{6 \pm \sqrt{11}}{10}$

(e) $\dfrac{1 \pm \sqrt{29}}{5}$

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22. Find the distance in the plane between the points $(\underset{x_1}{2},\underset{y_1}{1})$ and $(\underset{x_2}{3},\underset{y_2}{-1})$.

(a) 5

(b) $\boxed{\sqrt{5}}$

(c) 1

(d) $\pm\sqrt{5}$

(e) $\sqrt{27}$

Distance Formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-2)^2 + (-1-1)^2}$

$= \sqrt{1^2 + (-2)^2}$

$= \sqrt{1 + 4}$

$= \sqrt{5}$

---

23. Find the equation of the circle with center (-1,3) such that the point (5,3) is on the circle.

h k

Choices:

(a) $\boxed{(a)}$ $(x+1)^2 + (y-3)^2 = 36$
(b) $(x-1)^2 + (y+3)^2 = 36$
(c) $(x-5)^2 + (y-3)^2 = 6$
(d) $(x-5)^2 + (y-3)^2 = 4$
(e) $(x+1)^2 + (y-3)^2 = 16$

Circle center (h,k) with radius r

$(x-h)^2 + (y-k) = r^2$

So we have

$(x-(-1))^2 + (y-3)^2 = r^2$

$(x+1)^2 + (y-3)^2 = (\sqrt{36})^2$

$(x+1)^2 + (y-3)^2 = 36$

Radius of circle is the distance from center $(-1,3)$ to any point on the circle $(5,3)$

$r = \sqrt{(5-(-1))^2 + (3-3)^2}$

$= \sqrt{(5+1)^2 + 0^2}$

$= \sqrt{6^2 + 0}$

$= \sqrt{36}$

24. Solve the following equation for x.

$$x - 1 = \sqrt[2]{-2x + 2}$$

Choices:

(a) $x = \pm 1$
(b) The equation has no real solutions.
(c) $x = 2$ only.
(d) $\boxed{x = 1 \text{ only.}}$
(e) $x = \pm 3$

$(x-1)^2 = -2x+2$

$(x-1)(x-1) = -2x+2$

$x^2 - x - x + 1 = -2x + 2$

$x^2 \cancel{-2x} + 1 = \cancel{-2x} + 2$

$x^2 + 1 - 2 = 2 - 2$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x - 1 = 0$ or $x + 1 = 0$

$x = 1$ $\quad$ $x = -1$

Check $x=1$

$1 - 1 \stackrel{?}{=} \sqrt{-2(1)+2}$

$0 \stackrel{?}{=} \sqrt{-2+2}$

$0 = \sqrt{0}$ ✓

Check $x=-1$

$-1-1 \stackrel{?}{=} \sqrt{-2(-1)+2}$

$-2 \stackrel{?}{=} \sqrt{2+2}$

$-2 \stackrel{?}{=} \sqrt{4}$

$-2 \neq 2$

25. Which one of the following points is on the graph of the equation

$$x^2 - 3xy = -5?$$

Choices:

X (a) $(1, 3/2)$ $\quad 1^2 - 3(1)\left(\frac{3}{2}\right) = 1 - \frac{9}{2} = \frac{2}{2} - \frac{9}{2} = \frac{-7}{2} \neq -5$
X (b) $(0, 4)$ $\quad 0^2 - 3(0)(4) = 0 - 0 = 0 \neq -5$
(c) $\boxed{(1, 2)}$ $\quad 1^2 - 3(1)(2) = 1 - 6 = -5$ ☺
X (d) $(2, 1)$ $\quad 2^2 - 3(2)(1) = 4 - 6 = -2 \neq -5$
X (e) $(-1, 2/3)$ $\quad (-1)^2 - 3(-1)\left(\frac{2}{3}\right) = 1 + 2 = 3 \neq -5$