Name: ____ **Section:** ________

MA 109 Spring 2014

Exam 3 April 16, 2014

Directions:

- Do not remove this page—you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.
- The exam consists of multiple choice and short answer questions. Record your answers on this page by filling in the appropriate selection, for example:

  ● (B) (C) (D) (E).

- The exam is out of 100 total points: 5 points for each of 20 questions. **Only** this front page will be graded and **no partial credit** will be awarded. It is recommended that you check your work!

1. (A) (B) (C) ● (E)
2. (A) (B) ● (D) (E)
3. (A) ● (C) (D) (E)
4. (A) (B) (C) (D) ●
5. ● (B) (C) (D) (E)
6. (A) (B) ● (D) (E)
7. (A) (B) (C) ● (E)
8. ● (B) (C) (D) (E)
9. (A) (B) ● (D) (E)
10. (A) (B) (C) (D) ●
11. (A) (B) (C) (D) ●
12. (A) (B) (C) ● (E)
13. (A) (B) ● (D) (E)
14. (A) (B) ● (D) (E)
15. (A) ● (C) (D) (E)
16. (A) ● (C) (D) (E)
17. (A) (B) (C) ● (E)
18. (A) (B) ● (D) (E)
19. ● (B) (C) (D) (E)
20. (A) (B) (C) (D) ●

For grading use:

Total	
	(out of 100 pts)

Name: ____ **Section:** __________

Multiple Choice: Show your work in the space below and shade the correct answer on the front page for each of the following.

1. Choose the statement that best describes the following table.

input x	2	4	1	-3	5
output y	4	4	3	1	5

Choices:

(a) The table represents a function with domain $\{1, 3, 4, 5\}$ and range $\{-3, 1, 2, 4, 5\}$.

(b) The table does not represent a function.

(c) The table represents a one-to-one function.

$\boxed{\text{(d)}}$ The table represents a function with domain $\{-3, 1, 2, 4, 5\}$ and range $\{1, 3, 4, 5\}$

(e) The table represents a function with both domain and range all real numbers.

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2. For $f(x) = x^2 - 1$ and $g(x) = \sqrt{x-4}$, find a formula for the composition $g(f(x))$.

$g(f(x)) = \underline{g}(x^2-1) = \sqrt{(x^2-1)-4} = \sqrt{x^2-5}$

Choices:

(a) $x - \sqrt{5}$

(b) $x - 3$

$\boxed{\text{(c)}}$ $\sqrt{x^2 - 5}$

(d) 5

(e) $\sqrt{x-3}$

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3. Find the domain of the function $f(x) = \dfrac{1}{x-1}$.

Can <u>Not</u> have $x - 1 = 0$

$x - 1 + 1 = 0 + 1$

$x = 1$

Choices:

(a) $x = 1$

$\boxed{\text{(b)} \quad x \neq 1}$

(c) $x > 1$

(d) $x < -1$

(e) All real numbers.

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4. Let $f(x) = \sqrt{x-1}$. Which of the following is $f^{-1}(2)$?

Choices:

(a) 1
(b) 2
(c) $\sqrt{3}$
(d) 4
(e) 5

$f^{-1}(y) = x$ means $f(x) = y$
so $f^{-1}(2) = x$ means $f(x) = 2$
therefore $\sqrt{x-1} = 2$
$2^2 = x - 1$

$4 = x - 1$
$4 + 1 = x - 1 + 1$
$5 = x$
so $f(5) = 2$
which means
$f^{-1}(2) = 5$

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5. Find the inverse function of $f(x) = \dfrac{5x+1}{3}$.

Choices:

(a) $f^{-1}(x) = \dfrac{3x-1}{5}$
(b) $f^{-1}(x) = \dfrac{3y-1}{5}$
(c) $f^{-1}(x) = \dfrac{3}{5x+1}$
(d) $f^{-1}(x) = x$
(e) $f(x)$ does not have an inverse since it is not one-to-one.

$y = \dfrac{5x+1}{3}$
$x = \dfrac{5y+1}{3}$
$3x = 5y + 1$
$3x - 1 = 5y$

$\dfrac{3x-1}{5} = \dfrac{\cancel{5}y}{\cancel{5}}$
$y = \dfrac{3x-1}{5}$
so $f^{-1}(x) = \dfrac{3x-1}{5}$

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6. Find the domain of $(f \cdot g)(x)$ where $f(x) = \dfrac{1}{x}$ and $g(x) = x + 1$. Note: The operation is multiplication, not composition.

Choices:

(a) All real numbers.
(b) $x \neq 1$
(c) $x \neq 0$
(d) $x > 0$
(e) $x > -1$

Domain of f is $(-\infty, 0) \cup (0, \infty)$
Domain of g is $(-\infty, \infty)$
Common region is domain of $f \cdot g$ = $(-\infty, 0) \cup (0, \infty)$

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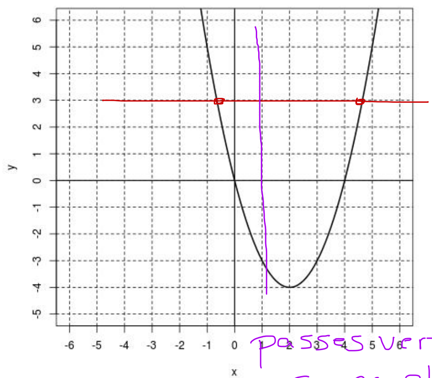
7. Which one of the following statements describes the graph below?



fails horizontal line test
so ~~not~~ a 1-1 function

passes vertical line test
so graph is a function

Choices:

~~(a)~~ The graph fails the vertical line test.

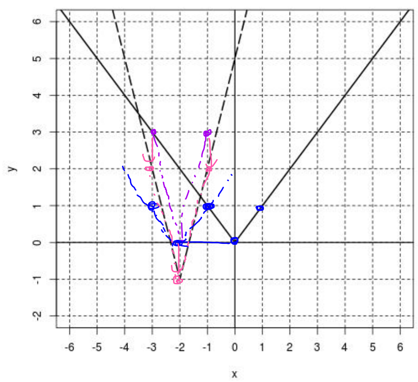
~~(b)~~ The graph does not represent a function.

~~(c)~~ The graph represents a one-to-one function.

$\boxed{\text{(d)}}$ The graph represents a function but not a one-to-one function.

~~(e)~~ The graph passes the horizontal line test.

8. In the picture below, the graph of $y = |x|$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$.



1^{st} Left 2 $\quad y = |x+2|$

2^{nd} Vertical Scale by 3 $\quad y = 3|x+2|$

3^{rd} Shift down by 1 $\quad y = 3|x+2| - 1$

Choices:

(a) $\boxed{g(x) = 3|x+2| - 1}$

(b) $g(x) = |3x| - 1$

(c) $g(x) = 3|x-2| - 1$

(d) $g(x) = 2|x+3| + 1$

(e) $g(x) = 3|x+2| + 1$

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9. For $f(x) = x^2 + x - 1$, find the difference quotient

$$\frac{f(x+h) - f(x)}{h}.$$

Choices:

(a) $2x + h^2 + 2$

(b) $\dfrac{x^2 + h + 2}{h}$

(c) $\boxed{2x + h + 1}$

(d) $2x + 1$

(e) $2x + h$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[(x+h)^2 + (x+h) - 1\right] - \left[x^2 + x - 1\right]}{h}$$

$$= \frac{x^2 + 2xh + h^2 + x + h - 1 - x^2 - x + 1}{h} = \frac{2xh + h^2 + h}{h}$$

$$= \frac{\cancel{h}(2x + h + 1)}{\cancel{h}} = \boxed{2x + h + 1}$$

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10. If $(6, -2)$ lies on the graph of $f(x)$, find a point on the graph of $y = g(x)$ if $g(x) = f(2x) + 4$.

Choices:

(a) (12,4)

(b) (12,-6)

(c) (2,4)

(d) (4,2)

(e) (3,2)

$f(6) = -2$ so we need $2x = 6$

$x = 3$

therefore $g(3) = f(2 \cdot 3) + 4$

$= f(6) + 4$

$= -2 + 4$

$= 2$ So g has the point $(3,2)$

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11. If a Johnny Jakes drives 30 miles from Lexington to Frankfort in 0.75 hours, stops 0.5 hours for lunch and then drives 60 miles from Frankfort to Louisville in one hour, what was his average speed from Lexington to Louisville?

30 miles — 60 miles

Lexington — Frankfort — Louisville

$\Delta t = 0.75$ hours — 0.5 hour stop — $\Delta t = 1$ hour

Choices:

(a) 90 miles.

(b) 90 miles/hr.

(c) 60.23 miles/hr.

(d) 51.43 miles/hr.

(e) 40 miles/hr.

average speed $= \dfrac{\Delta d}{\Delta t} = \dfrac{30+60}{0.75+0.5+1} = \dfrac{90}{2.25} = 40$ mph

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12. Find the average rate of change of the function $f(x) = x^2 - x + 1$ as x changes from $x = 1$ to $x = 2$.

Choices:

(a) 1

(b) h

(c) $x + h$

(d) 2

(e) -2

$\dfrac{f(b)-f(a)}{b-a} = \dfrac{f(2)-f(1)}{2-1} = \dfrac{3-1}{2-1} = \dfrac{2}{1} = 2$

$f(2) = 2^2 - 2 + 1 = 4 - 2 + 1 = 2 + 1 = 3$

$f(1) = 1^2 - 1 + 1 = 1 - 1 + 1 = 1$

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13. For $f(x) = 2x - 2$ and $g(x) = x^2 - 1$, find $\left(\frac{f}{g}\right)(5)$.

Choices:

(a) $\frac{-1}{3}$

(b) 2

$\boxed{\text{(c)} \quad \frac{1}{3}}$

(d) 3

(e) -3

$$\left(\frac{f}{g}\right)(5) = \frac{f(5)}{g(5)} = \frac{2(5)-2}{5^2-1} = \frac{10-2}{25-1} = \frac{8}{24} = \frac{1}{3}$$

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14. Which one of the graphs of the following equations represents a one-to-one function?

Choices:

~~(a)~~ $y = x^2 - x + 1$ ← parabola not one-to-one

~~(b)~~ $y = |x|$ ← V-shape not one-to-one

$\boxed{\text{(c)}}$ $y = x^3 + 5$ S-shape one-to-one

~~(d)~~ $x^2 + y^2 = 4$ circle not one-to-one

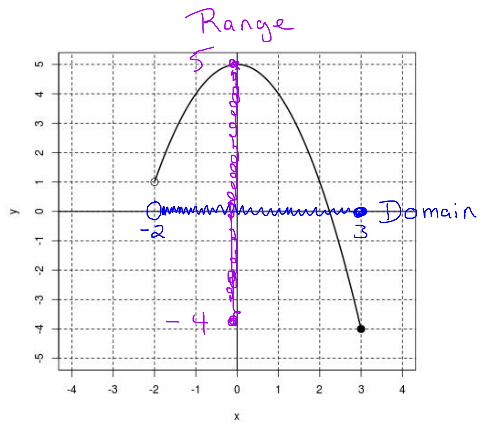
~~(e)~~ $y^2 - 2 = x$

$y^2 = x + 2$

$y = \pm\sqrt{x+2}$ not a function

7

15. The graph of $y = f(x)$ is depicted below. What is the domain and the range of $f(x)$?



Choices:

(a) The domain is $[-2, 3]$ and the range is $[-4, 5]$.

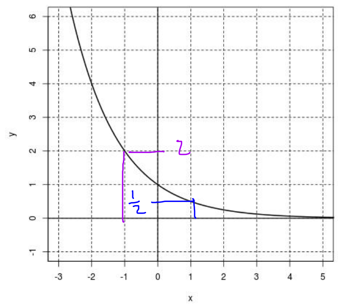
$\boxed{\text{(b)}}$ The domain is $(-2, 3]$ and the range is $[-4, 5]$.

(c) The domain is $(-4, 5]$ and the range is $[-4, 1]$.

(d) The domain is $(-2, 3]$ and the range is $(-4, 5)$.

(e) The domain is $[-4, 1]$ and the range is $(1, 3]$.

16. What is the formula for the graph of function below?



Note $f(1)=\frac{1}{2}$

and $f(-1)=2$

Choices:

(a) $y = 2^x$

(b) $y = \left(\frac{1}{2}\right)^x$

(c) $y = x^2$

(d) $y = 4^x$

(e) $y = (-2)^x$

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17. Which one of the following statements is true?

Choices:

(a) The average rate of change of a line depends on the starting point.

(b) To calculate the average rate of change of a function, you only need to know the starting point.

(c) The average rate of change of a function can not be calculated from the difference quotient.

(d) The average rate of change of a line is its slope.

(e) Average rate of change of all functions between any two points is constant.

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18. Find $f(5)$ for

$$f(x) = \begin{cases} x^2 & \text{if } x < 5 \\ x & \text{if } \boxed{5 \le x < 7} \\ 2x - 1 & \text{if } 7 \le x \end{cases}$$

$5 < 5$ False
$5 \le 5 < 7$ True
$7 \le 5$ False

Choices:

(a) 25
(b) 9
(c) 5
(d) 4
(e) 7

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19. If a culture starts out with 1000 bacteria and doubles every 3 hours, how many bacteria are there after 12 hours?

P_0 3

$P(t) = P_0 a^t$

$P(t) = 1000 a^t \longrightarrow P(t) = 1000 \left(2^{\frac{1}{3}}\right)^t$

$2000 = P(3) = 1000 a^3$ $\quad P(t) = 1000 \; 2^{t/3}$

$\frac{2000}{1000} = \frac{1000 a^3}{1000}$ $\quad P(12) = 1000 \cdot 2^{12/3} = 1000 \cdot 2^4 = 1{,}000 \cdot 16$

$2 = a^3$ $\quad = \boxed{16{,}000}$

$\sqrt[3]{2} = \sqrt[3]{a^3}$

$\sqrt[3]{2} = a$ or $a = 2^{\frac{1}{3}}$

Choices:

(a) 16,000
(b) 6,000
(c) 4,096
(d) 4,000
(e) 24,000

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20. A colony of bacteria grows exponentially according to the following data. Find a formula for the number of bacteria f as a function of the number of days x.

Day	0	1	2	3	4
Population	100	400	1,600	6,400	25,600

$P_0 = 100$

$f(x) = 100 a^x$

$400 = f(1) = 100 a^1$

$\frac{400}{100} = \frac{100 a}{100}$

$4 = a$

So $f(x) = 100 \cdot 4^x$

Choices:

(a) $f(x) = 100(6^x)$
(b) $f(x) = 100^x$
(c) $f(x) = 100(x^4)$
(d) $f(x) = 100(2^x)$
(e) $f(x) = 100(4^x)$
